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Formal Software Development

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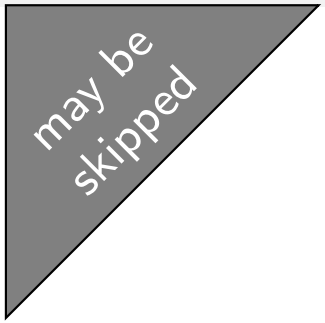
Introduction

This is an overview of the 'Formal Software Development' program. It is written for software professionals, and as such, it assumes some familiarity with symbolic manipulation.

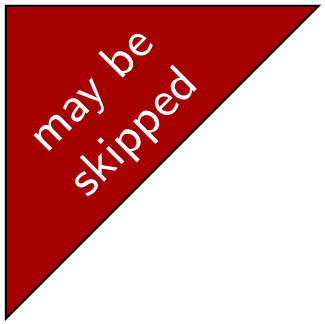
In this overview, we:

- Explain what a mathematical formal system is
- Show why formal systems are the foundation of software
- Give some concrete examples of formal software development
- Present the structure and the goals of the program

Introduction



Some slides are marked with this gray sticker; they contain some fairly elementary mathematical concepts, essentially *induction*. These slides may be skipped.



Other slides are marked with this red sticker; they assume knowledge of higher level algebra and treat *induction* at a more advanced level. These slides may also be skipped.

We include these optional slides because the program itself is a progression from basic concepts to more advanced ones, and we want to give all prospective students a bird's-eye view of the entire program and a roadmap to follow. Moreover, *induction is the central proof technique of the program*.

The use of mathematical logic

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Formal software development is the use of *mathematical logic* to specify, design, build, analyze and test software.

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- understand how to apply these tools to software engineering

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- *At the core of this use of logic are the formal systems*

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- We are not concerned for now with a *specific meaning* of such a system

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- Otherwise parts of the overview will sound gibberish

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- A language has a grammar: a way to specify which formulas are acceptable; the technical term is well-formed formulas, *wffs* in short.
- The rules of the grammar are known as formation rules, as opposed to the rules of system, which are known as *rules of inference*.

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- Similarly, aa*bb, aba*bbbbb, baba*abab are wffs (think of what X and Y are in these cases)
- Whereas **, *aba*, a*b*b are not wffs, because we cannot find any X and Y to fit the grammar requirements

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A finite sequence of deductions is called a *proof*. If a proof uses no premises other than axioms, then every wff in the sequence of the proof is a *theorem*. In particular, axioms are theorems, their proofs being sequences of length 1. Although the rules can be used to build proofs from any premises, we will not be using this facility in these slides. For us, proofs begin with axioms and dish out theorems.

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- metamathematics (study of formal systems themselves, from the outside)

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Proof. We show that b* is not a theorem. The axiom * has no b on the left. No rules allow the introduction of a b on the left of the *. □

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- Only one thing is left unclear: *what exactly is an algorithm?*

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This is not a theorem, it is a thesis, so there is no question of proving it. It captures the belief that humanity has nailed the concept of computability, that there exists no other mechanism that deserves to be called an algorithm.

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- *In one sentence, formal systems have computability built-in*

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So, for any theory that can be described algorithmically (and that's all we really need, but that's philosophy), whether a mathematical theory, a software theory, a physics theory, etc . . . , there is a formal system that can describe it.

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- If this sounds difficult, think of the way natural numbers or lists are introduced in your favorite functional language. The constructors embody this potential infinity.

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- The more expressive the system (i.e. the more powerful its language and rules of inference are), the less likely it is that it has such a procedure

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may be skipped

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Metatheorem: Dans is a decidable system

The proof is by induction. *Proofs by induction are typical of the metamathematics of formal systems*, that's why we include one here. They will be seen over and over during the program. Those unfamiliar with mathematical induction may skip the proof.

We treat induction at length in the course 'Formal semantics of programming languages'. *We also give a more detailed presentation of induction and recursion, as they relate to categories, in a more advanced section of this overview*. For those who need a quick brush-up, we include the definition of mathematical induction and one typical example of its use.

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may be skipped

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Let $P(n)$ be a predicate on the set $\mathbb{N}$ of natural numbers. Then

$$(P(0) \wedge (\forall k \in \mathbb{N}.P(k) \rightarrow P(k+1))) \rightarrow \forall n \in \mathbb{N}.P(n)$$

A typical proof by mathematical induction

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5. But $\sum_{i=0}^{k+1} i = \frac{(k+1)(k+2)}{2}$ is exactly $P(k+1)$
6. By mathematical induction, the formula (1) is true for all $n \in \mathbb{N}$ □

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  - a single *
  - only a's on the left, only b's on the right
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- We will prove that all theorems have this property. The metaproof is by induction on the length of the object proof.

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- Let $p_1, \ldots, p_{n+1}$ be a proof of length $n+1$. Then p_{n+1} must have been deduced from p_n by one of the rules 2,3, or 4. Now since the proof of p_n has length n, by the induction hypothesis, p_n has property P. But then p_{n+1} must also have property P because rules 2,3, and 4 all preserve this property. □

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- The most important proof technique of metamathematics
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- Induction itself can be formalized, for example, inside one of the most basic formal systems, Primitive Recursive Arithmetic (PRA), a sublogic of the better-known Peano Arithmetic

The central role of induction

- The most important proof technique of metamathematics
- (we already saw some of its uses)
- Induction itself can be formalized, for example, inside one of the most basic formal systems, Primitive Recursive Arithmetic (PRA), a sublogic of the better-known Peano Arithmetic
- We study PRA in the 'Main Concepts of Logic' course

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  - algebraic data is defined inductively (and it is known that *any computational form of data can be described algebraically*)

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- Some software development can indeed be based on PRA and inductive provers
- We study them in the 'Lisp and Inductive Provers' course
- Best known such prover: ACL2

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- (why is trust an issue? Because the more powerful the logic, the easier it is to become inconsistent)

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- Sometimes a *logic* does not include a proof calculus, it includes only a semantics!

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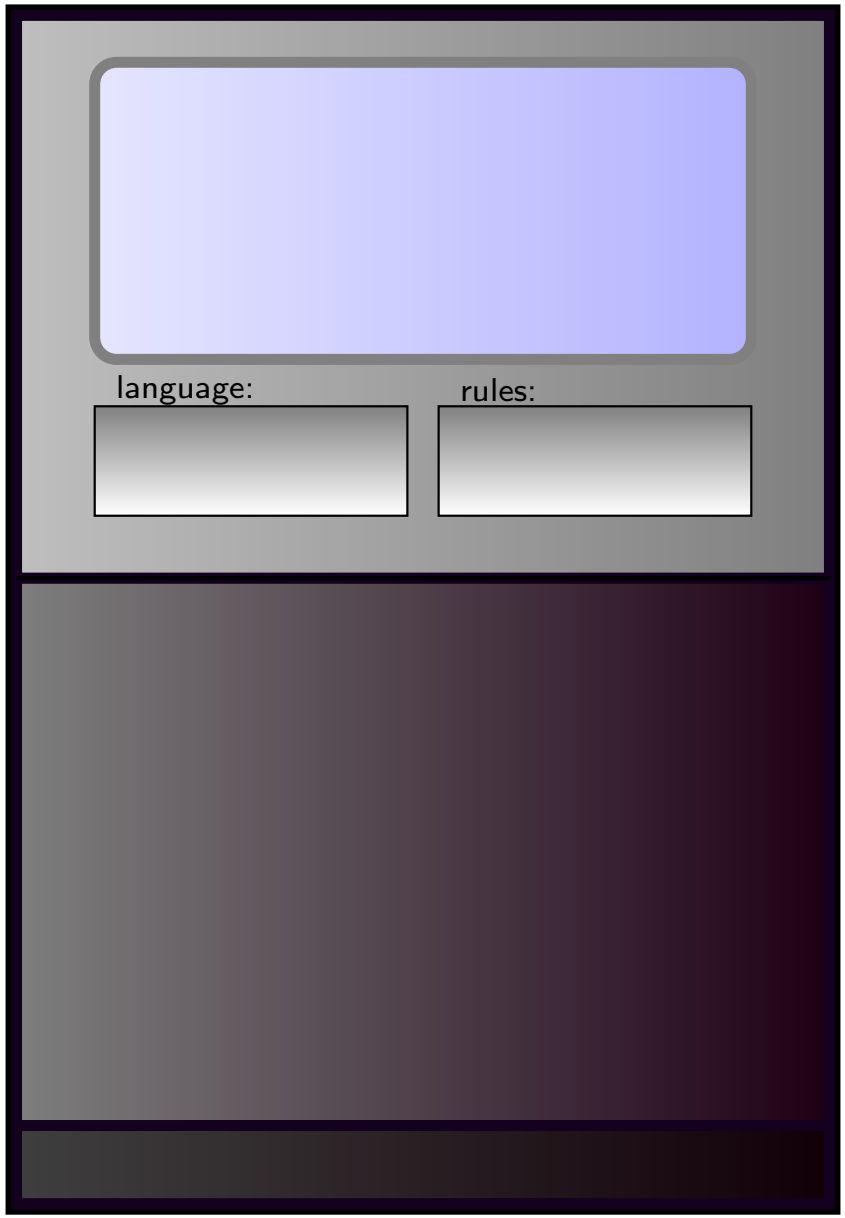
- By the way, logic for us is always *mathematical* logic. Otherwise, logic is a field of philosophy and we have no use for that.

Machines



Machines

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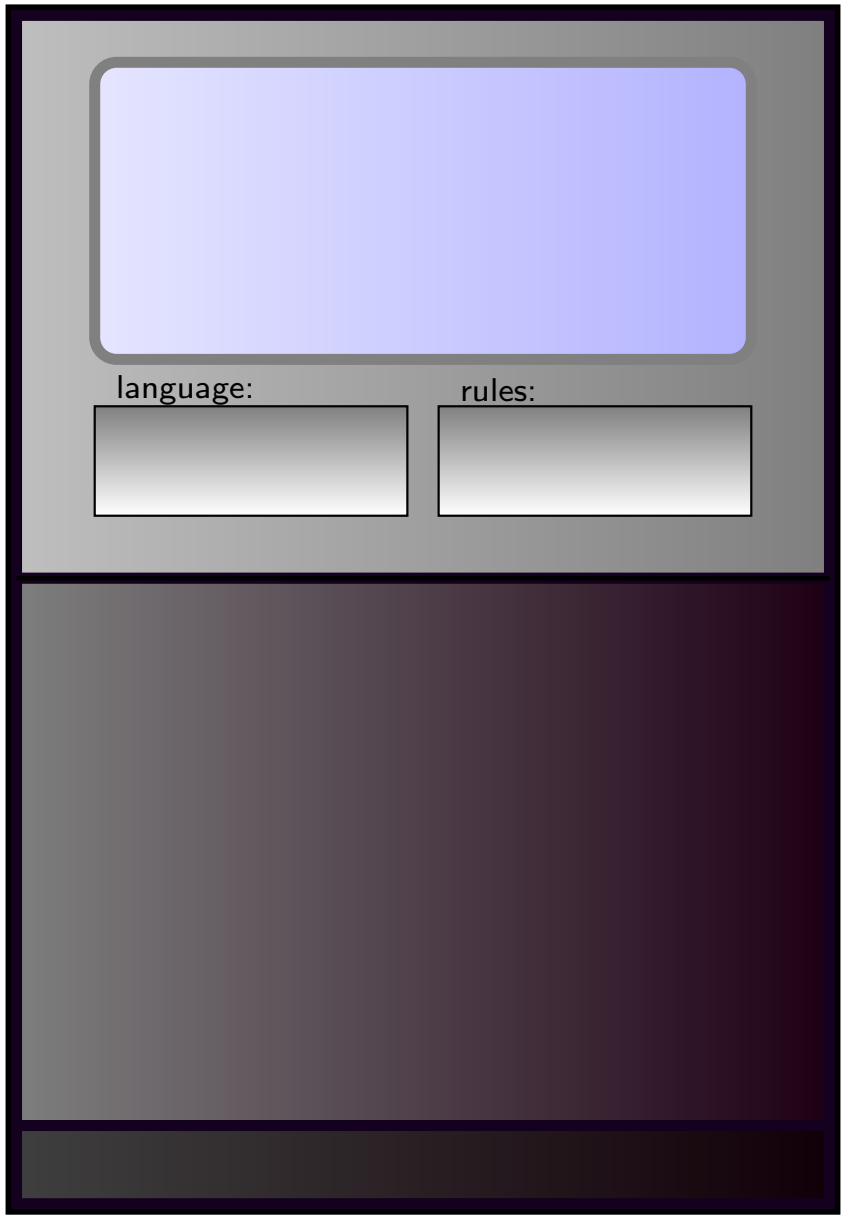

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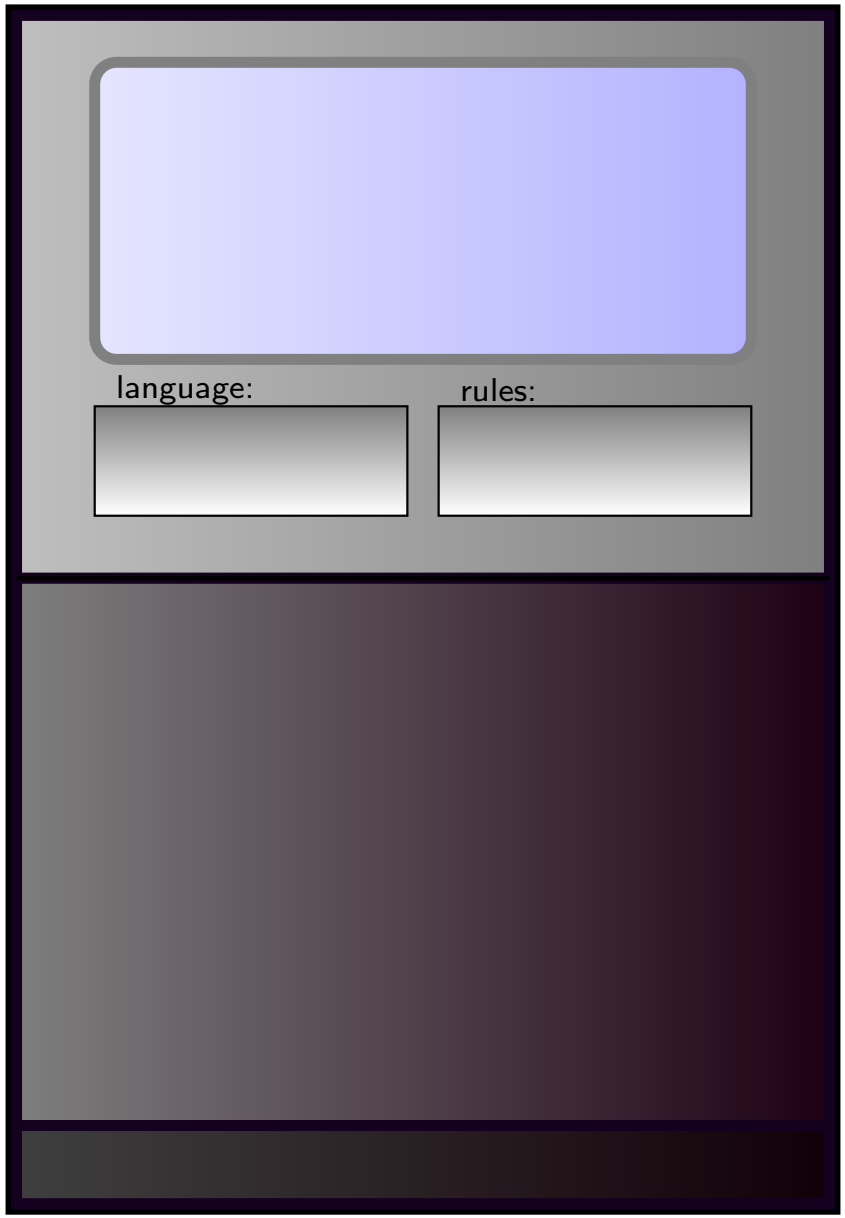

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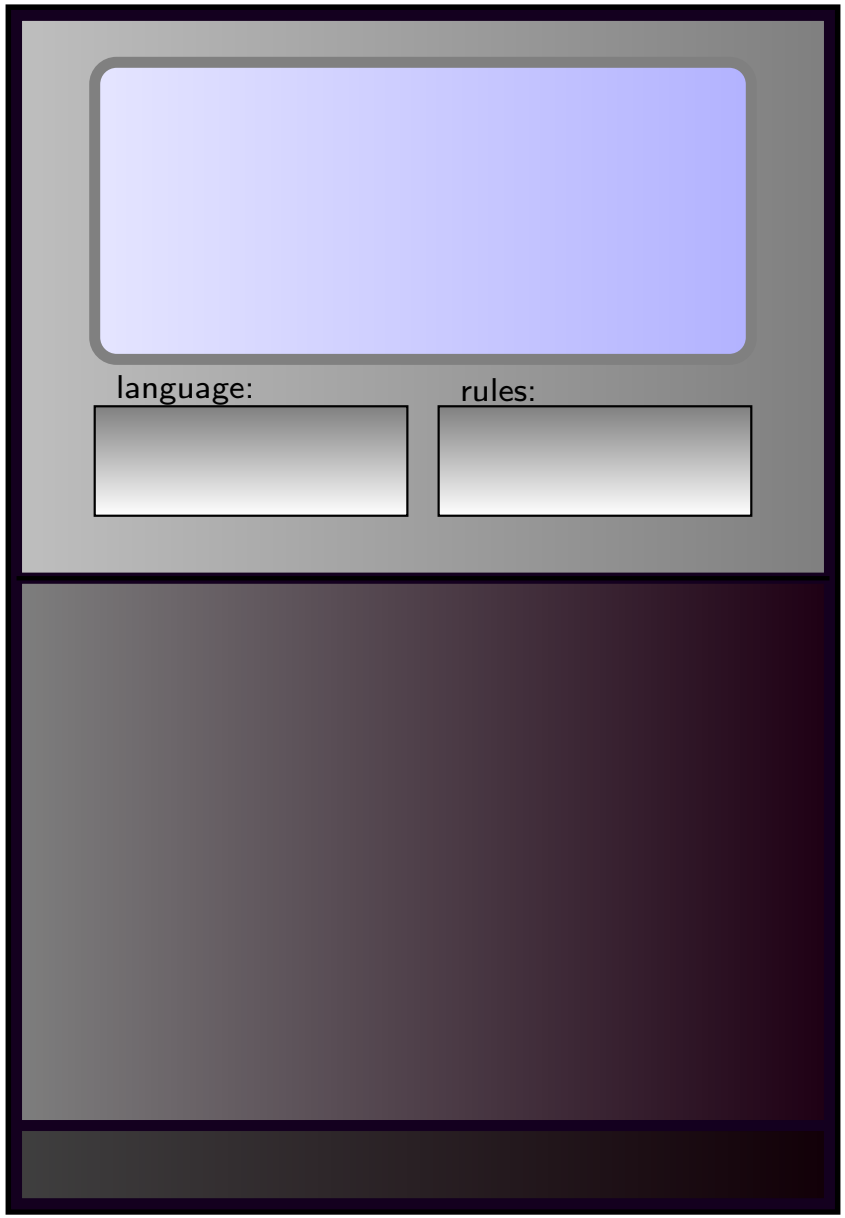

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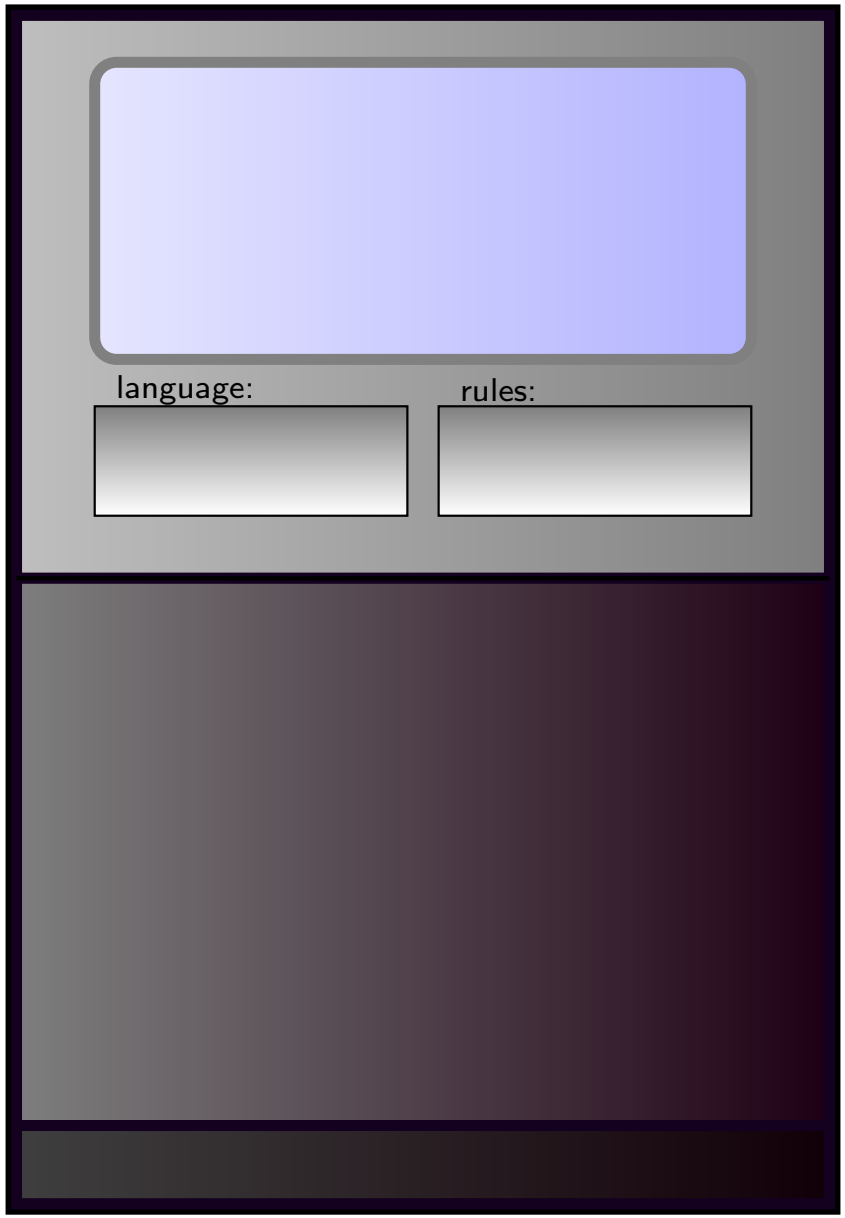

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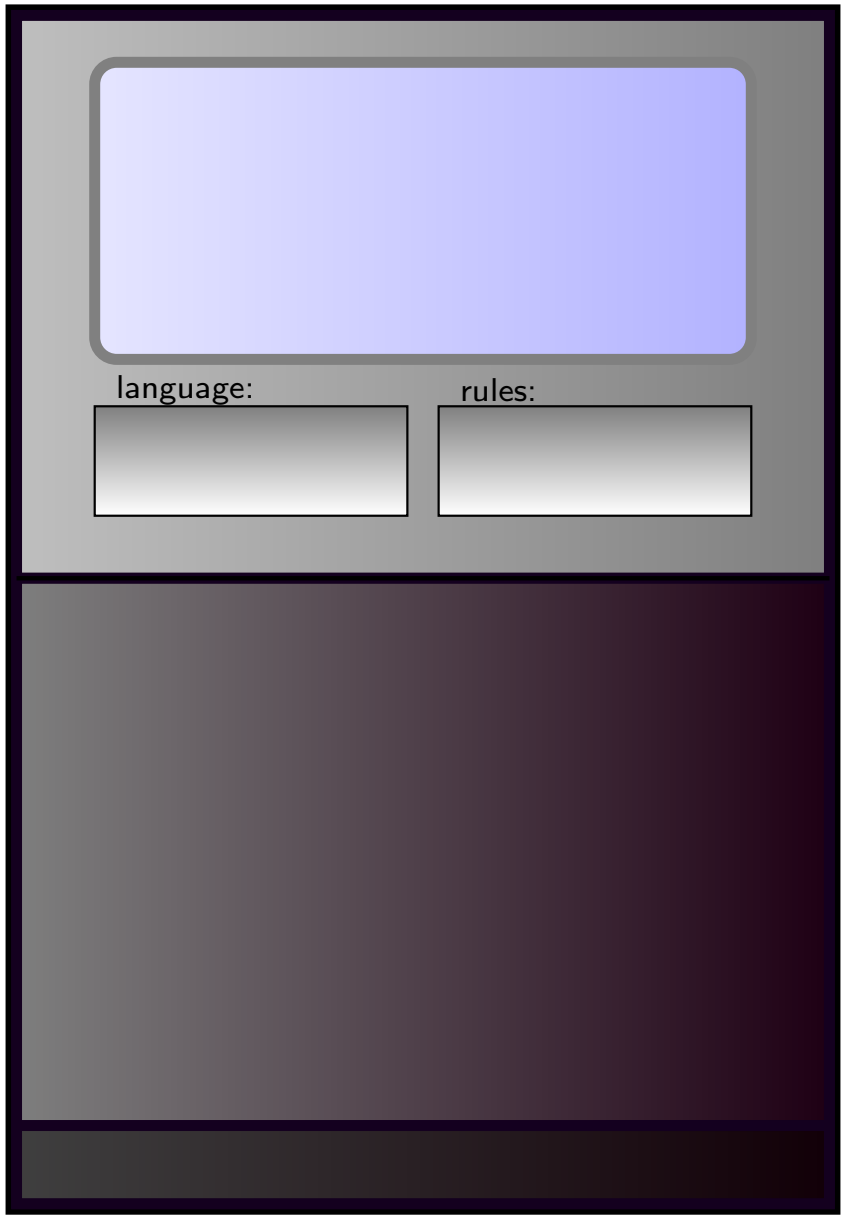

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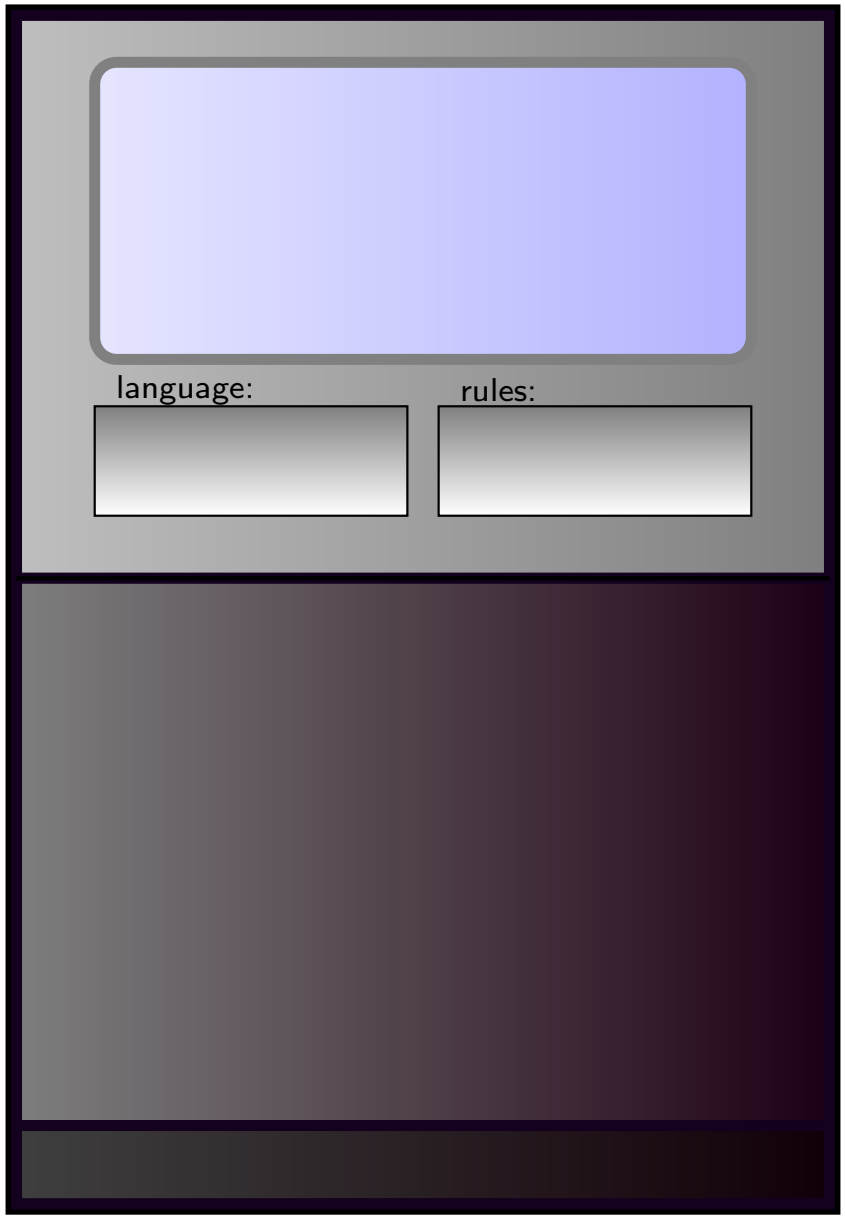

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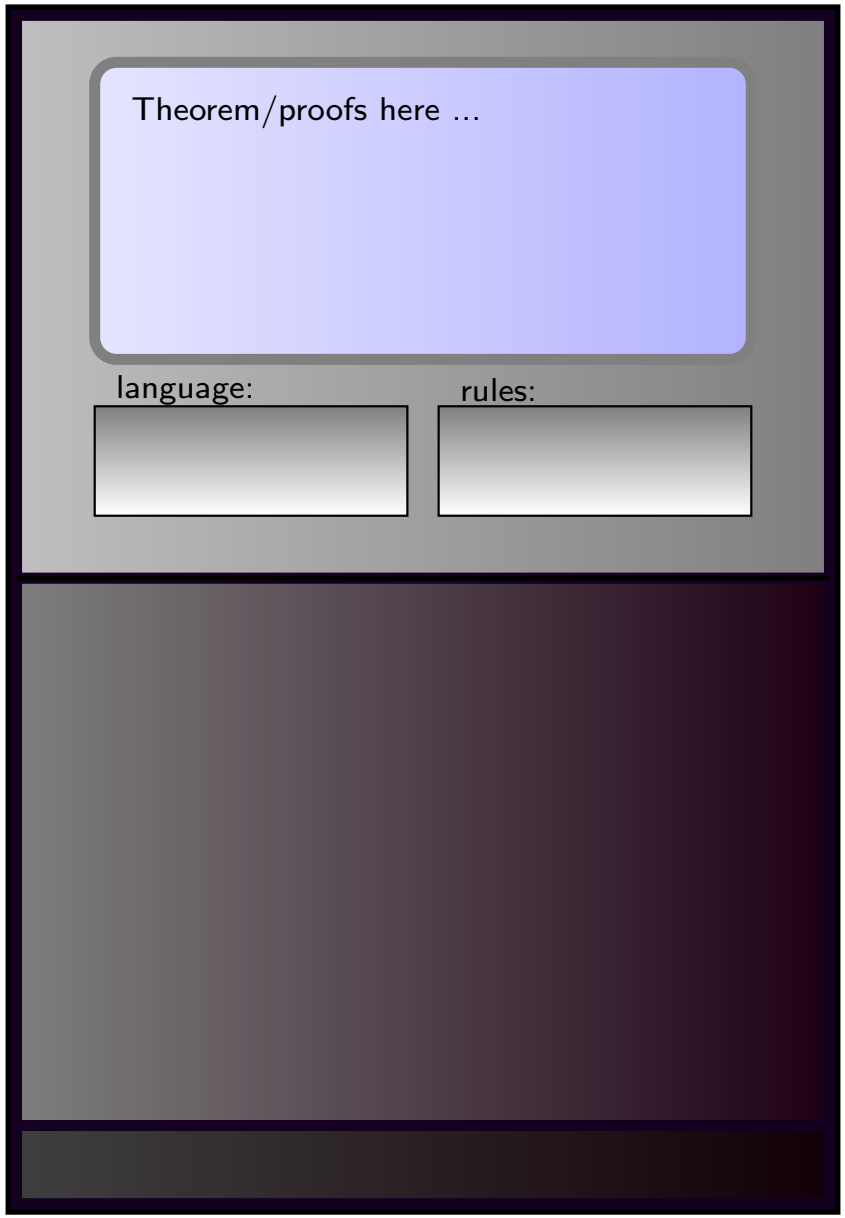

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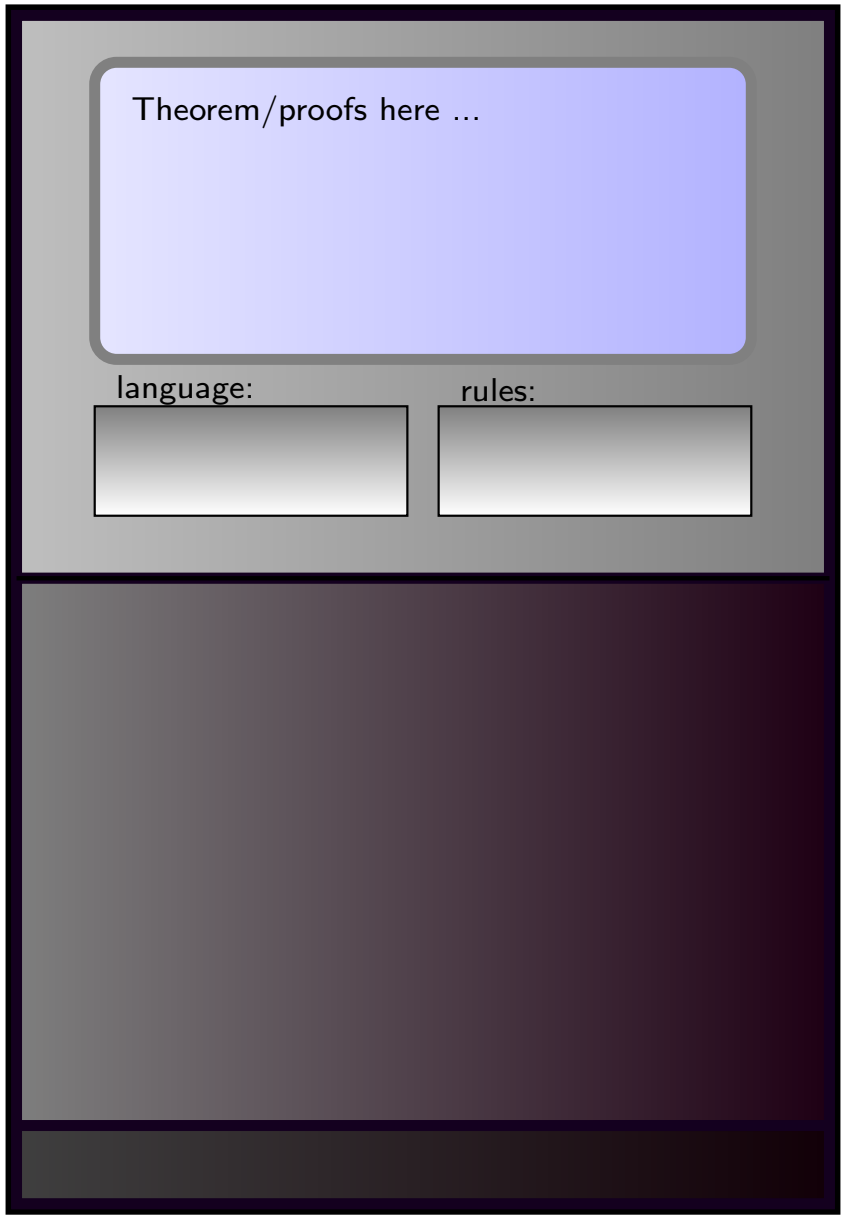
- The top is the 'object level' of the machine



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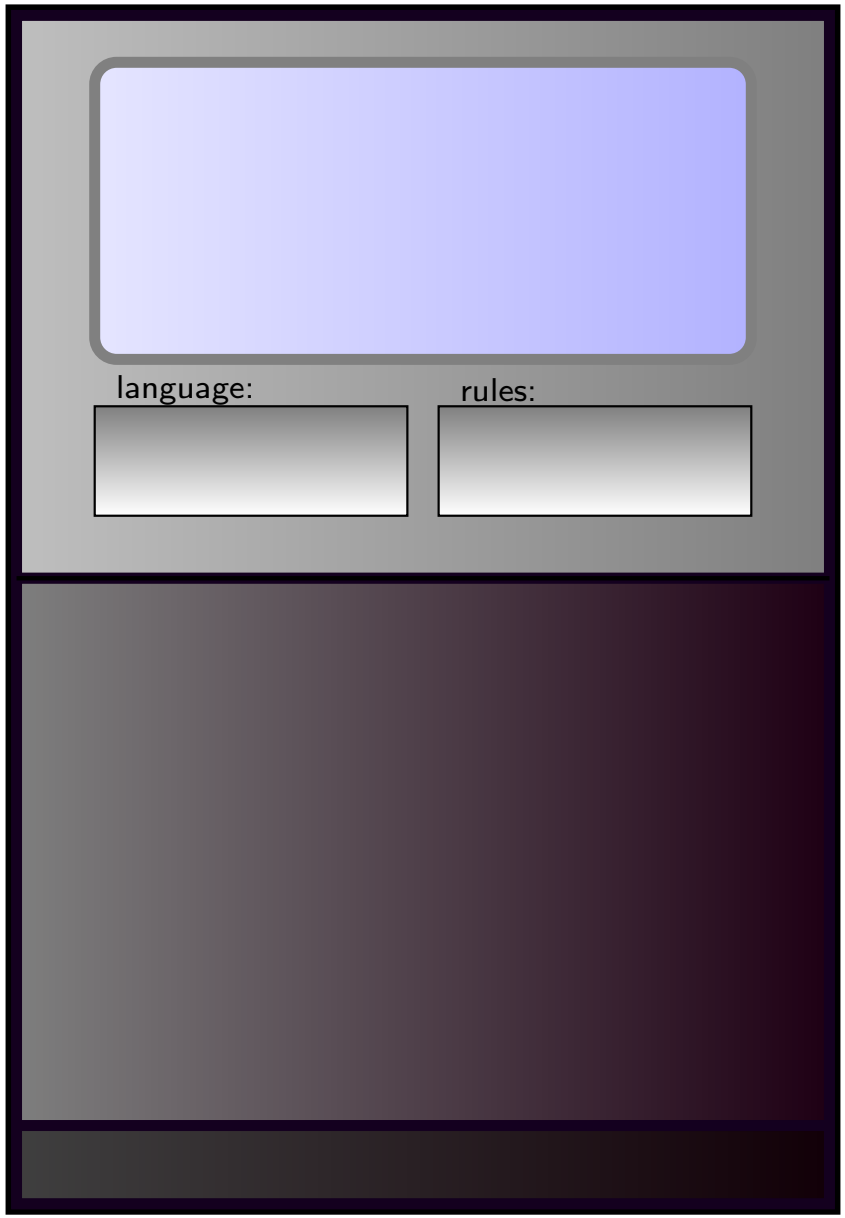

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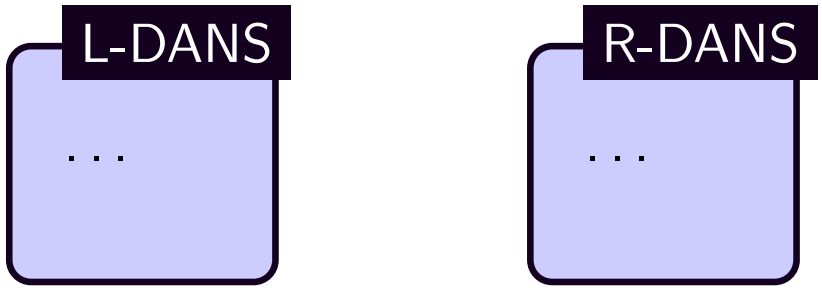

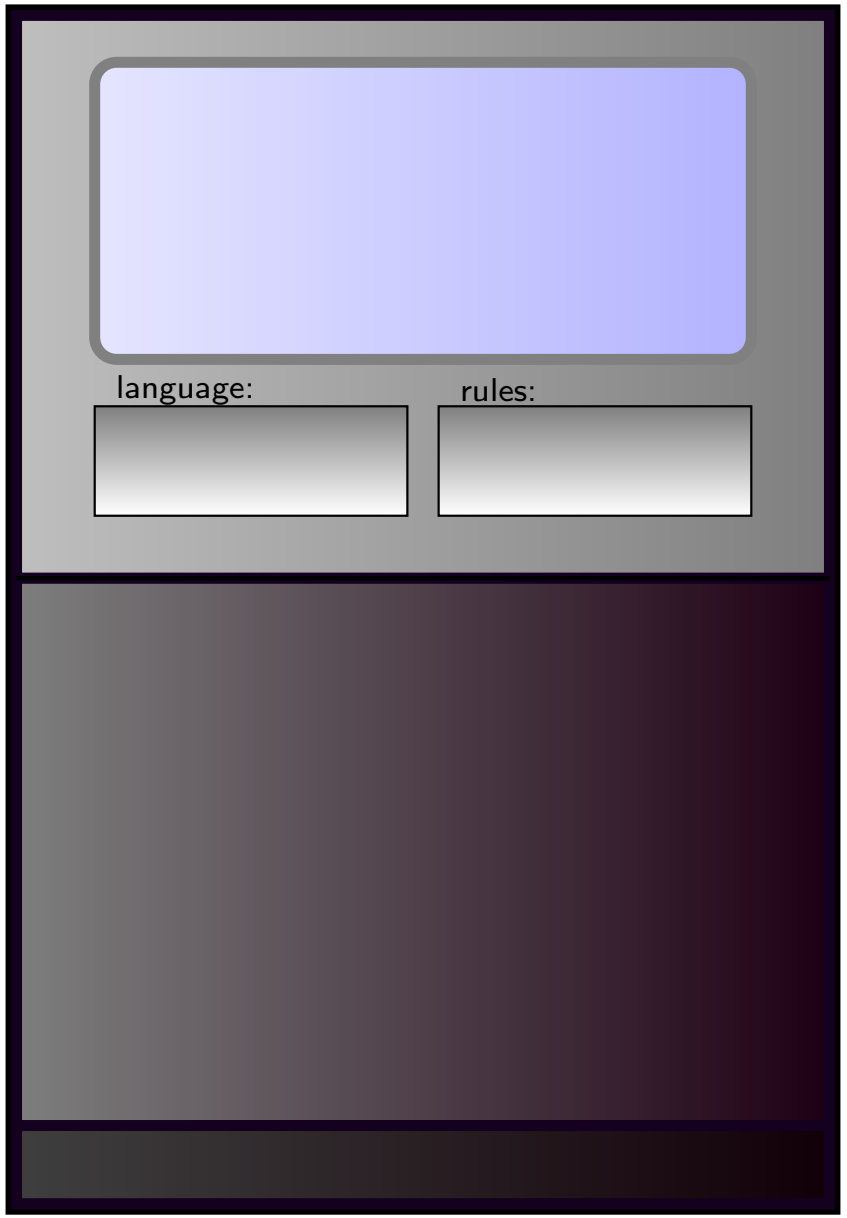

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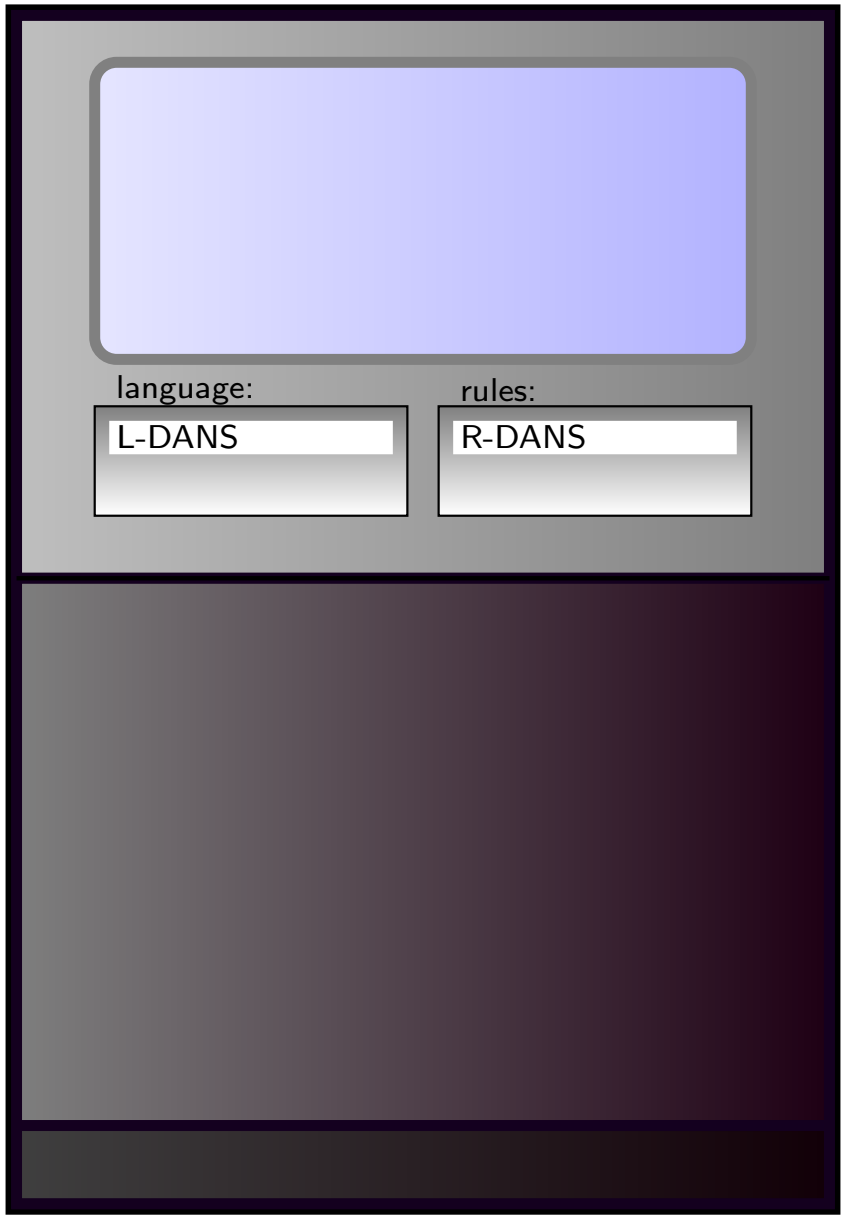

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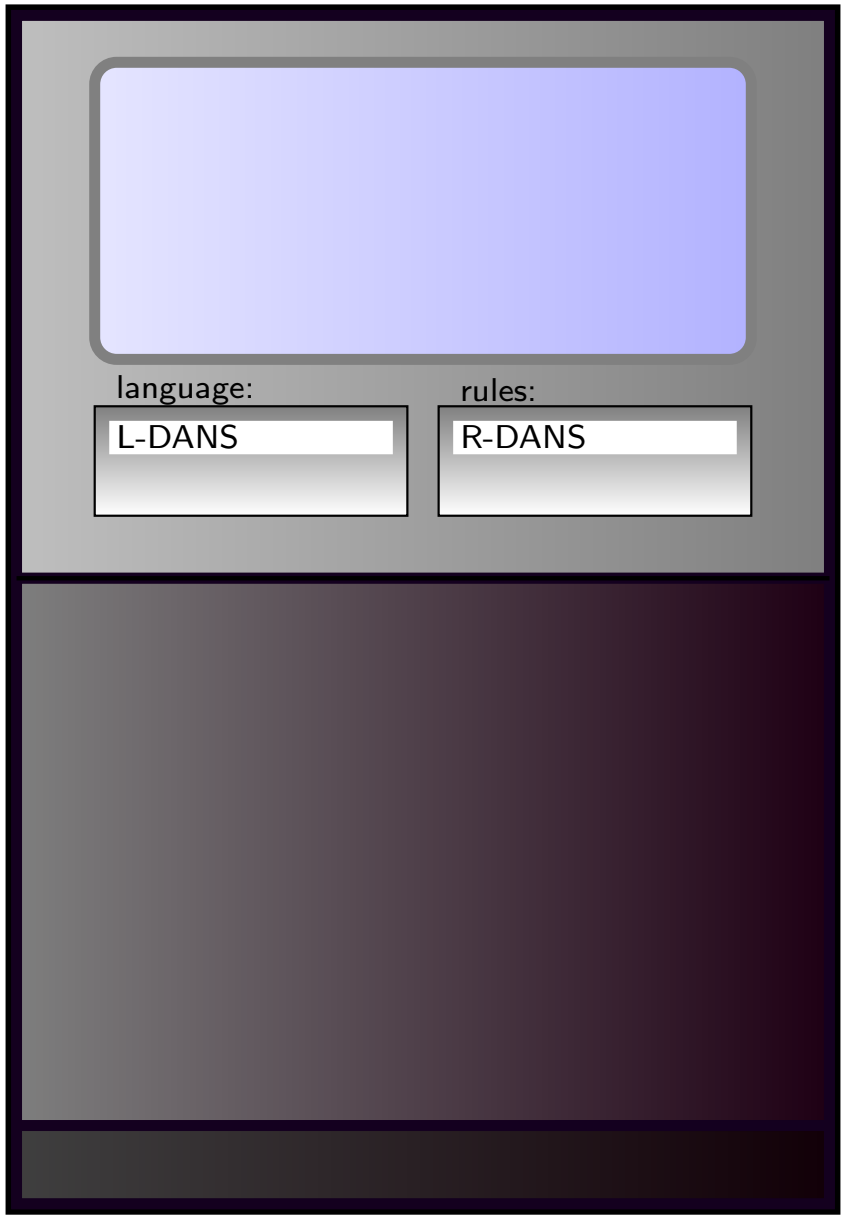

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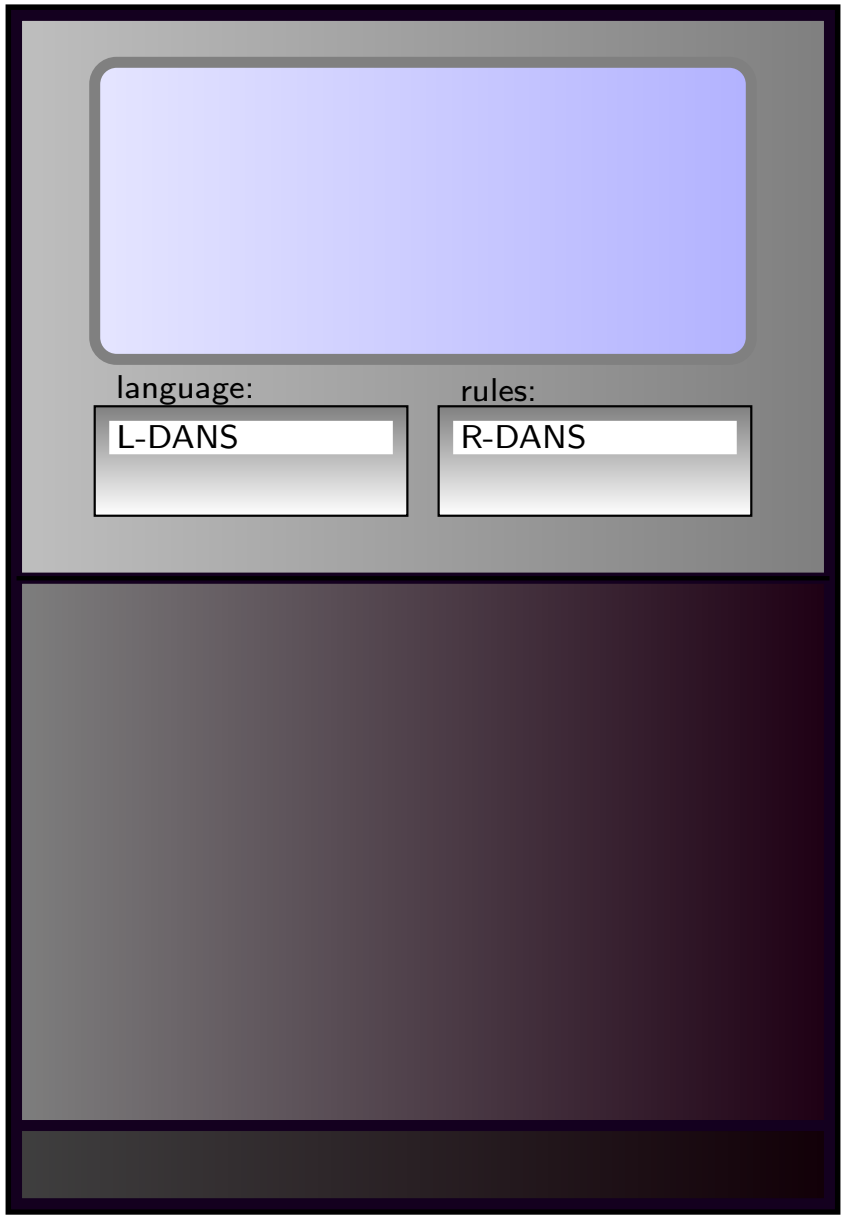

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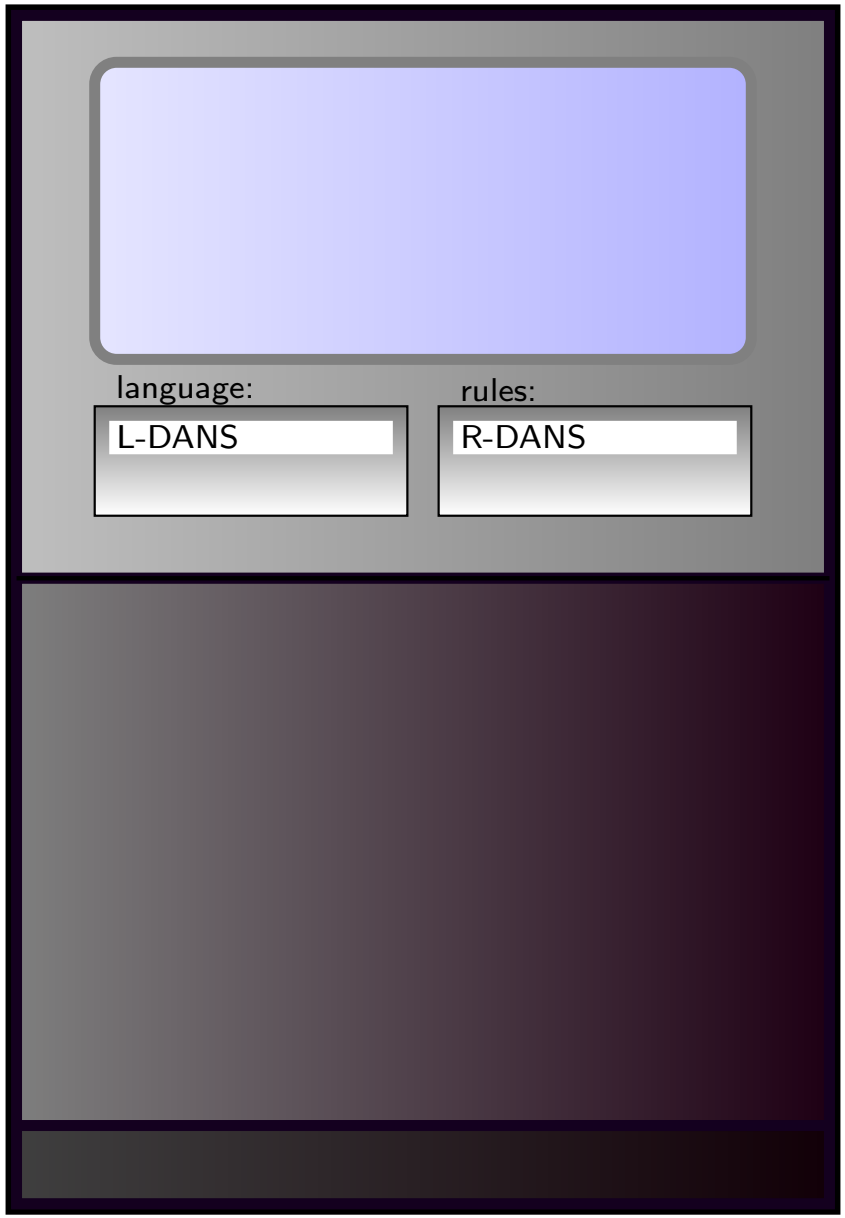

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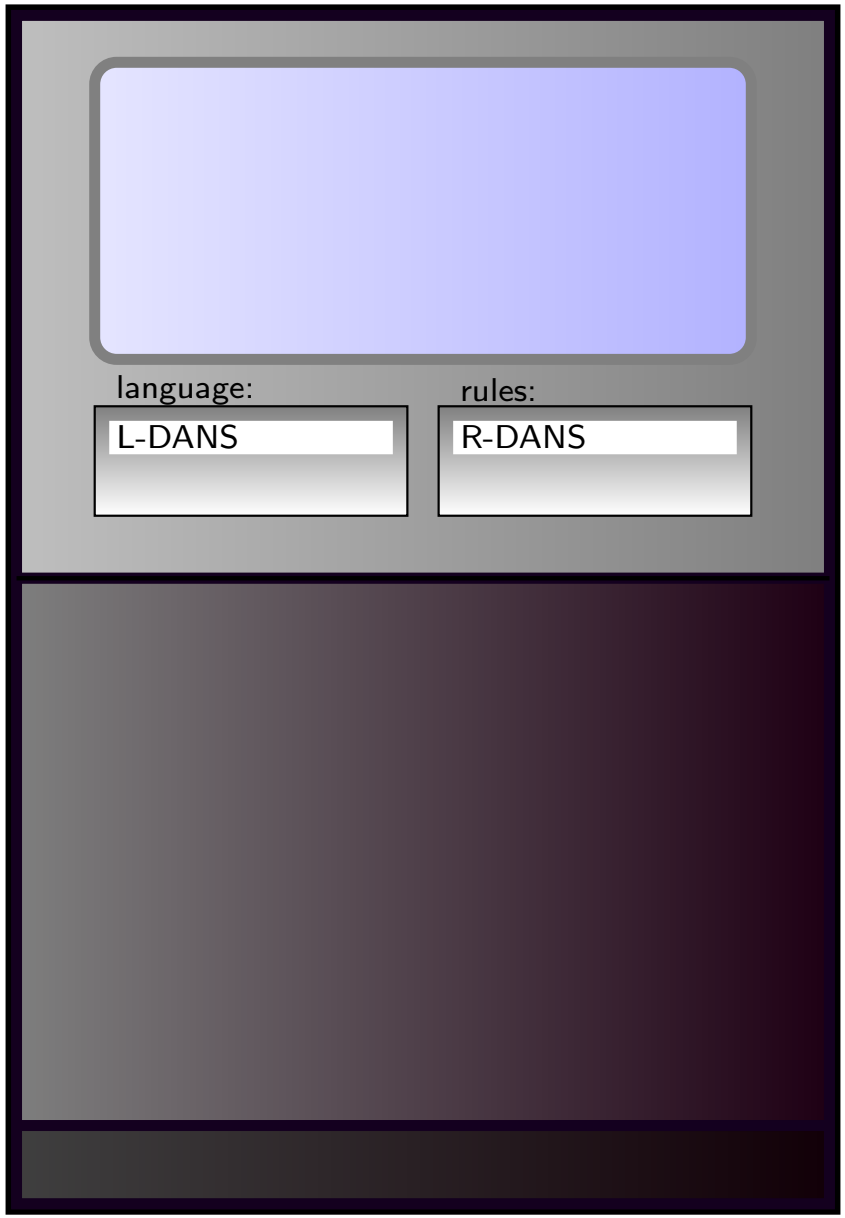

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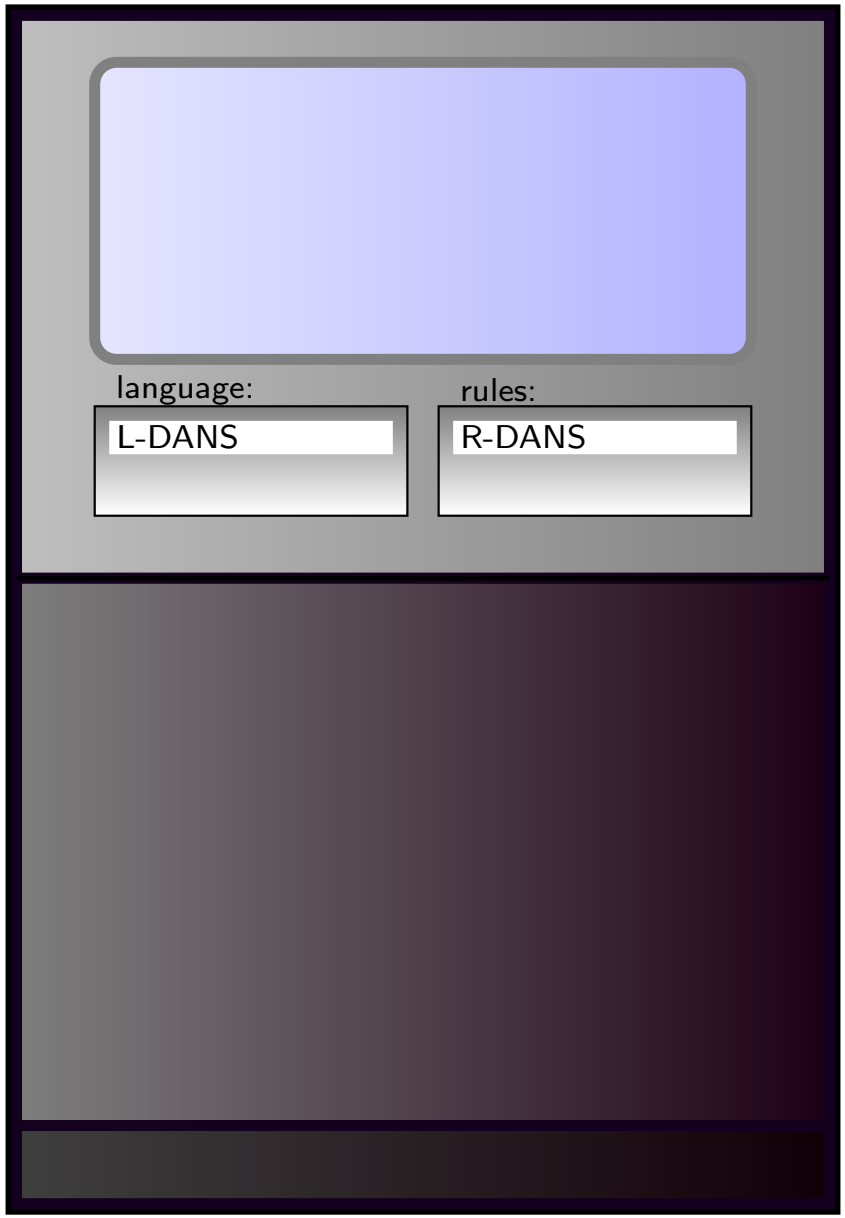

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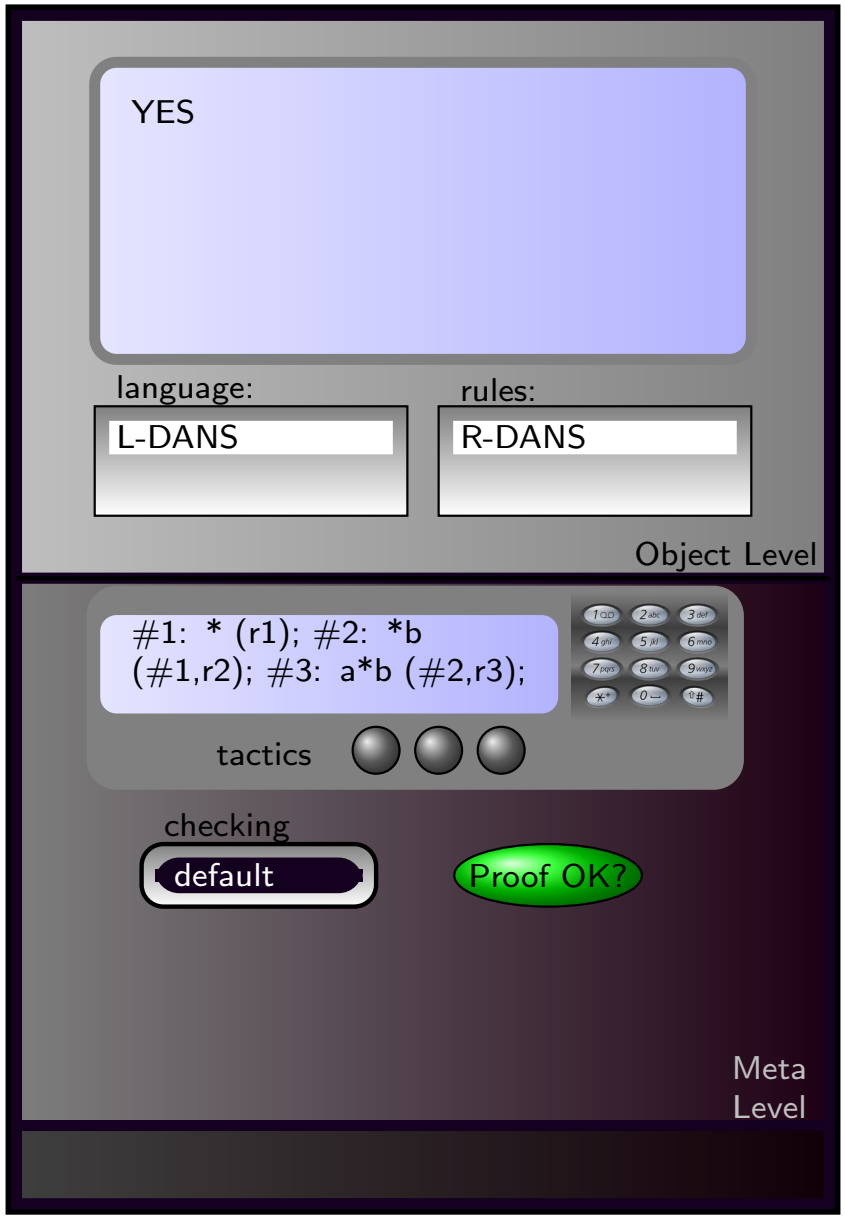

Proof checking algorithms

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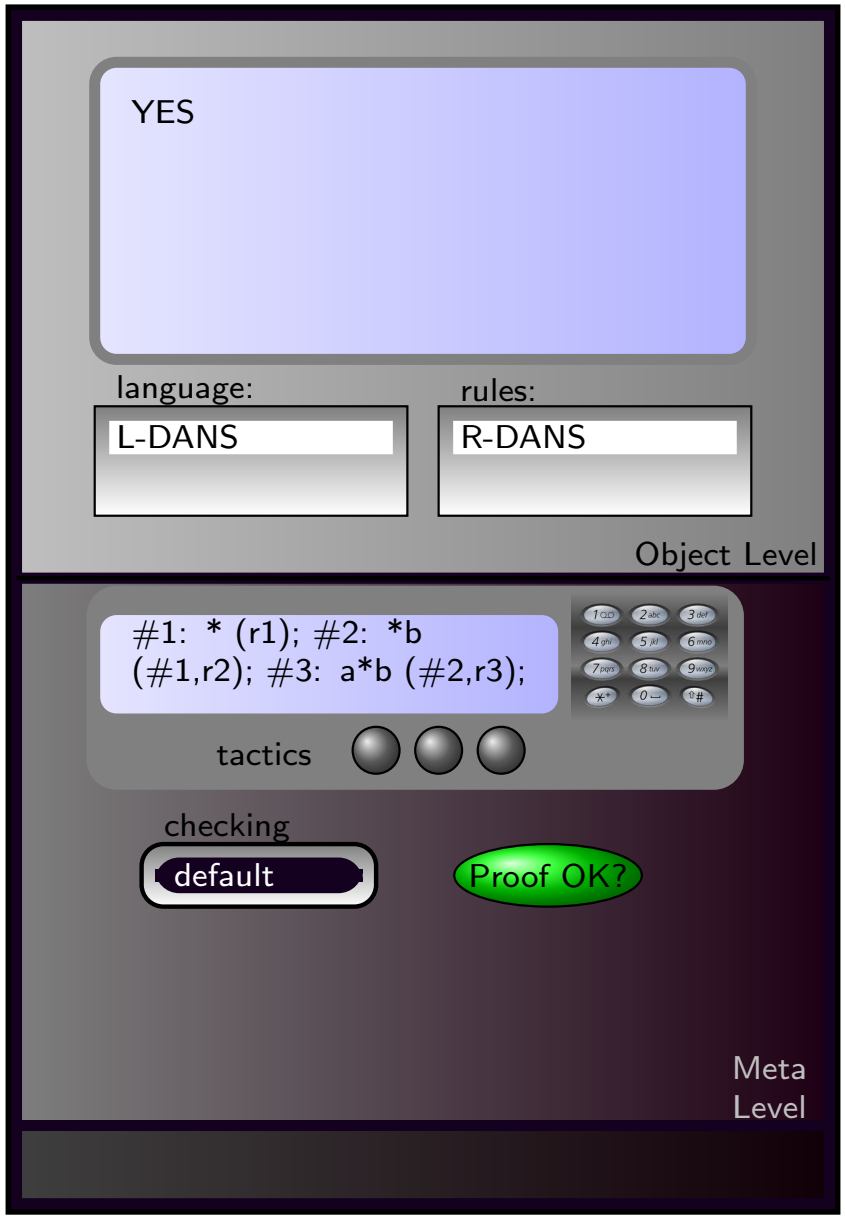

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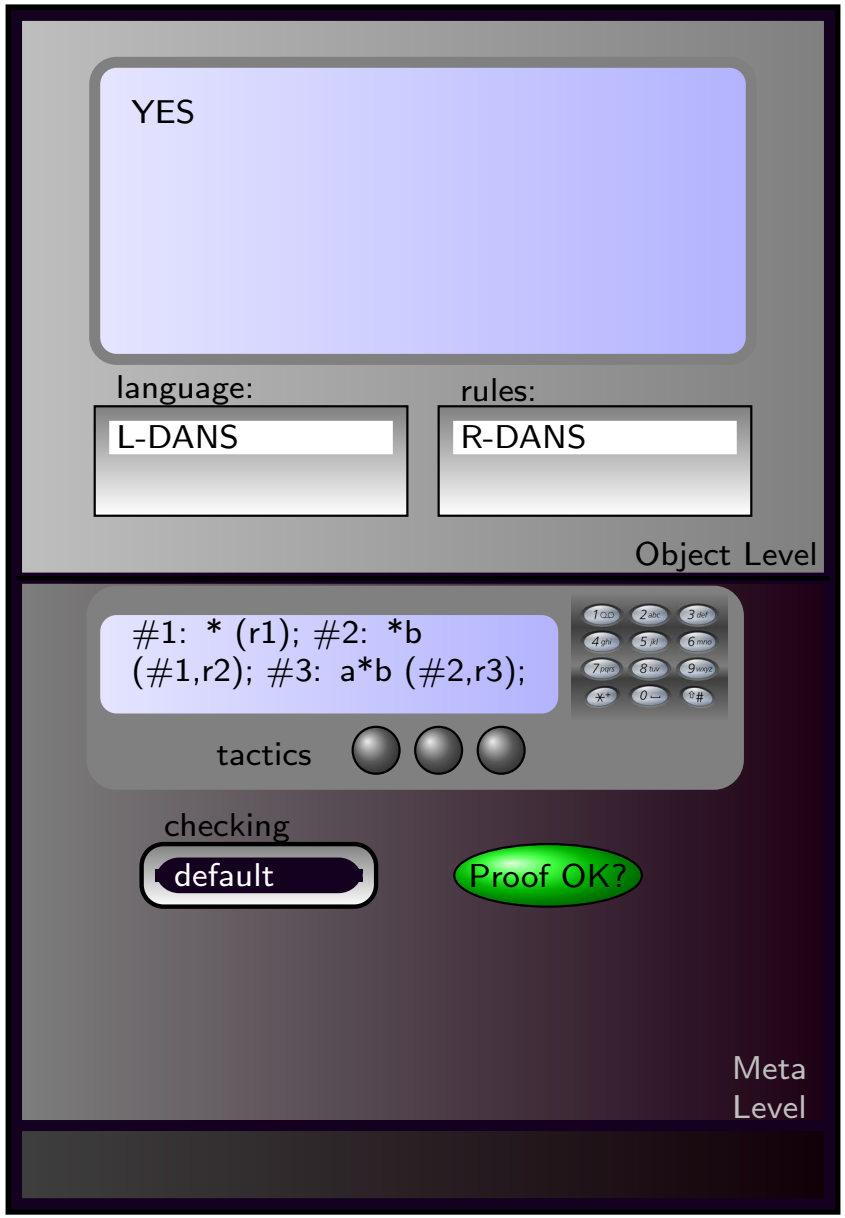

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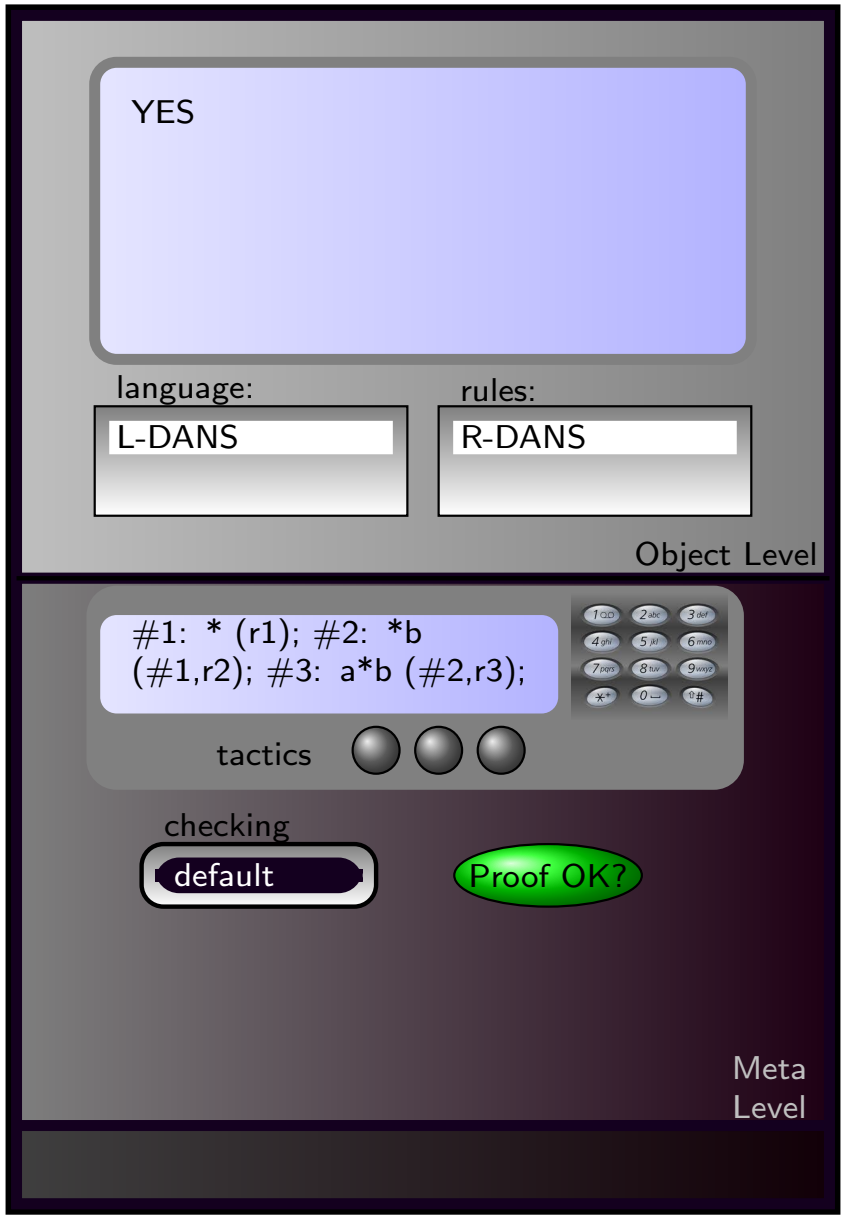

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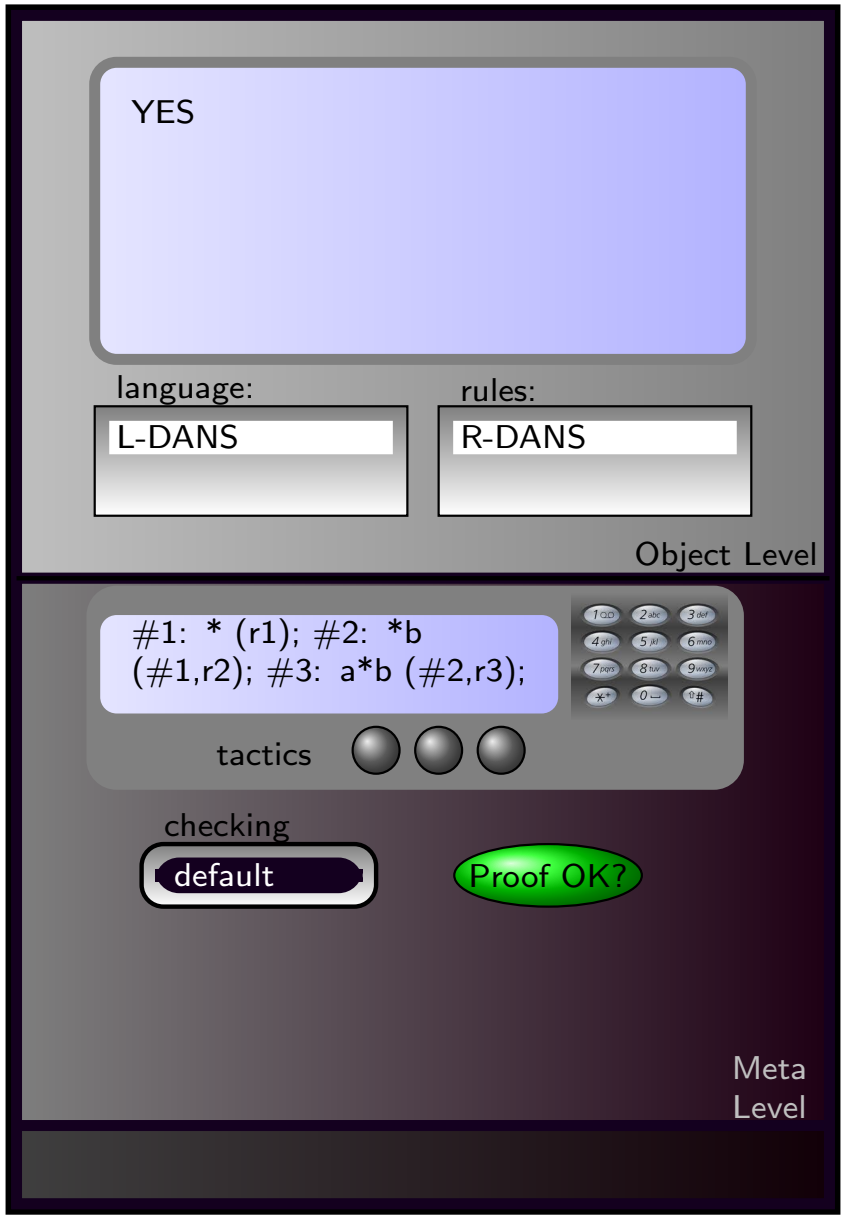

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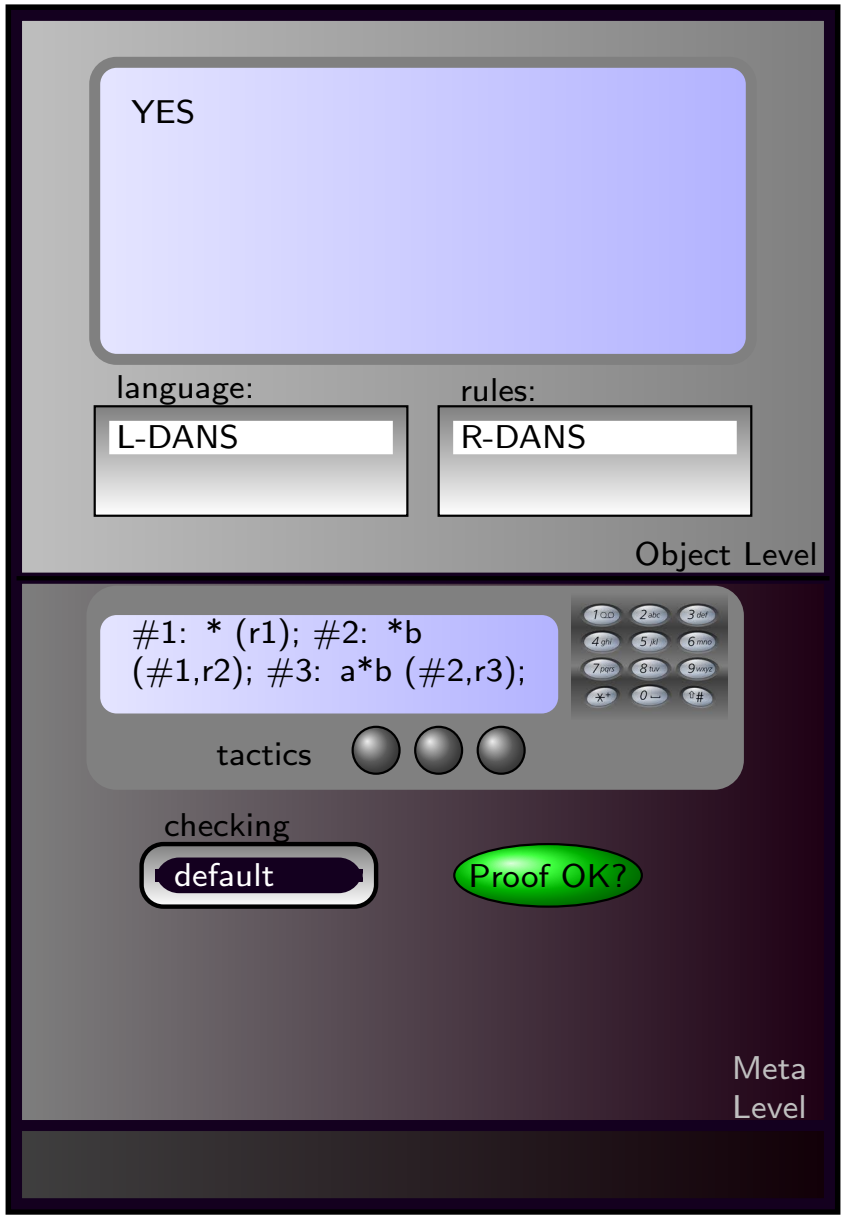

Proof construction with tactics

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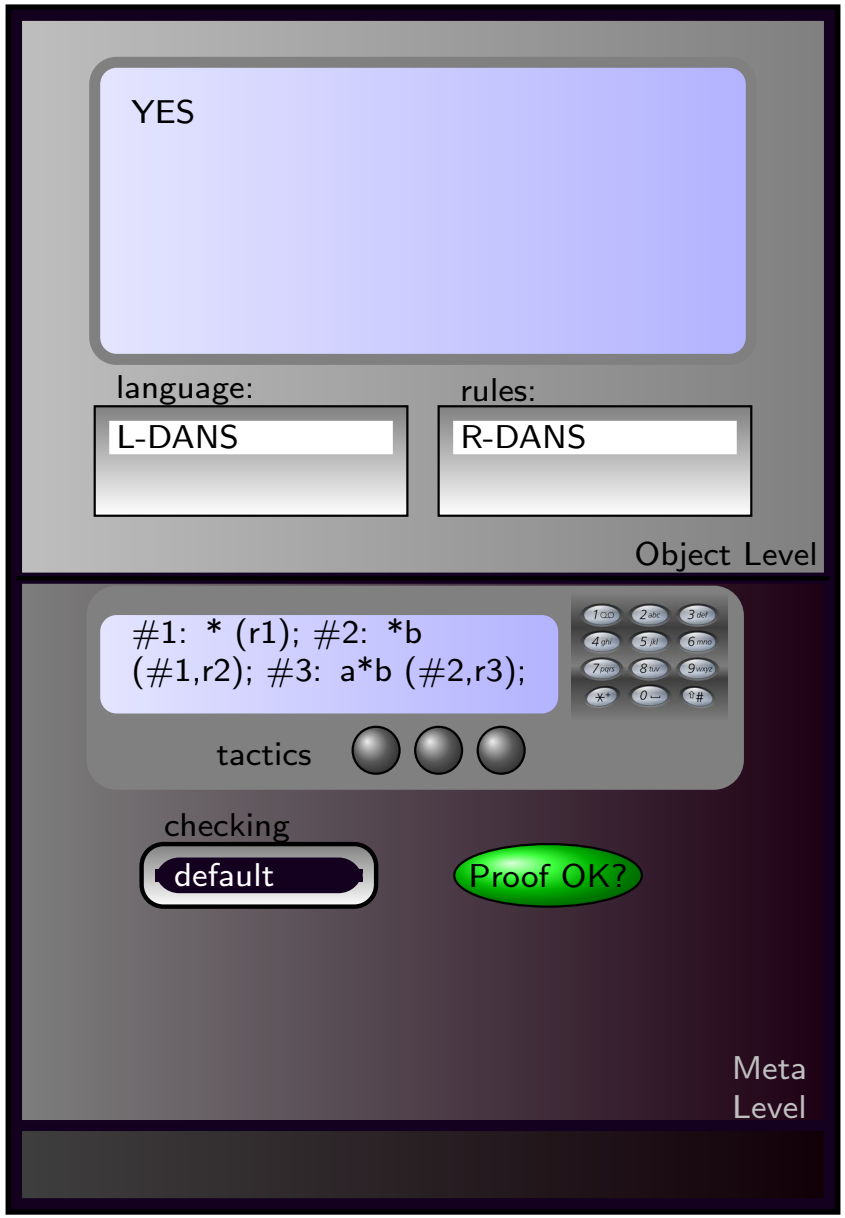

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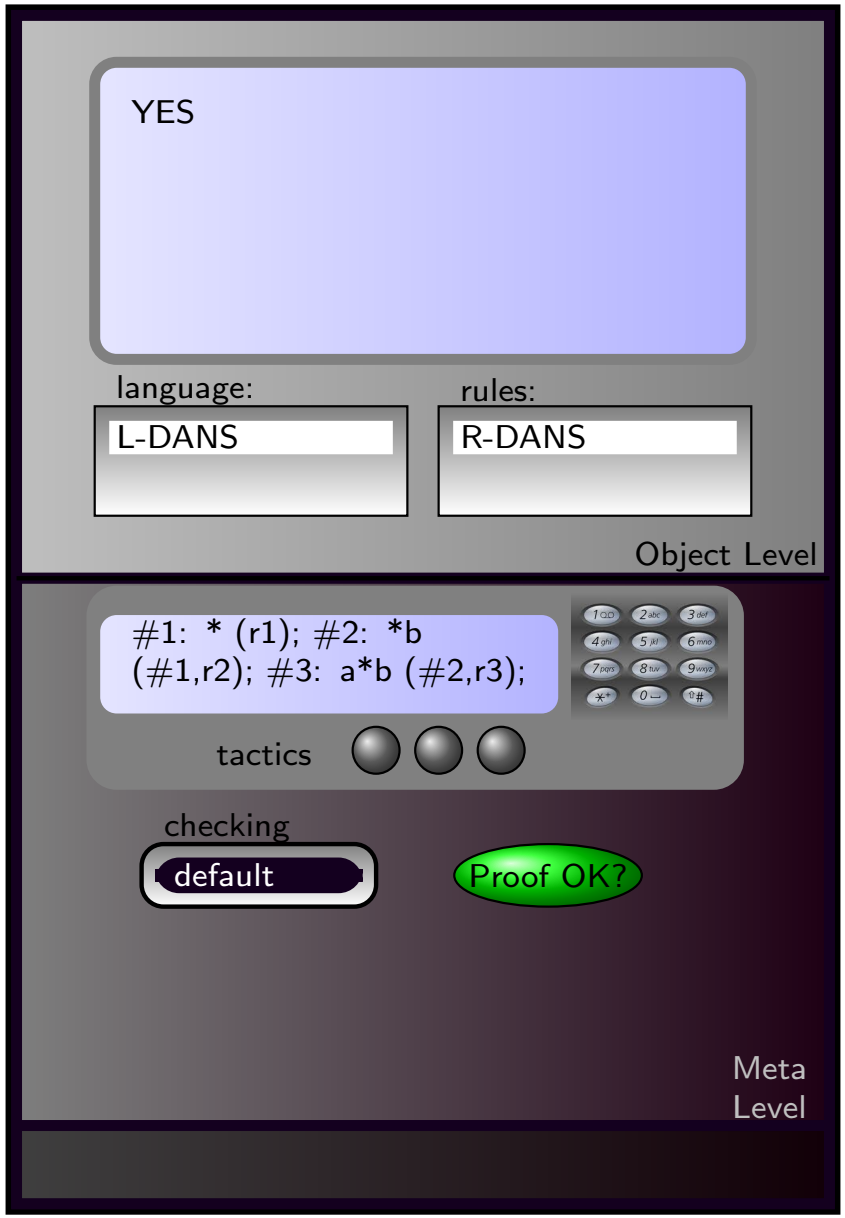

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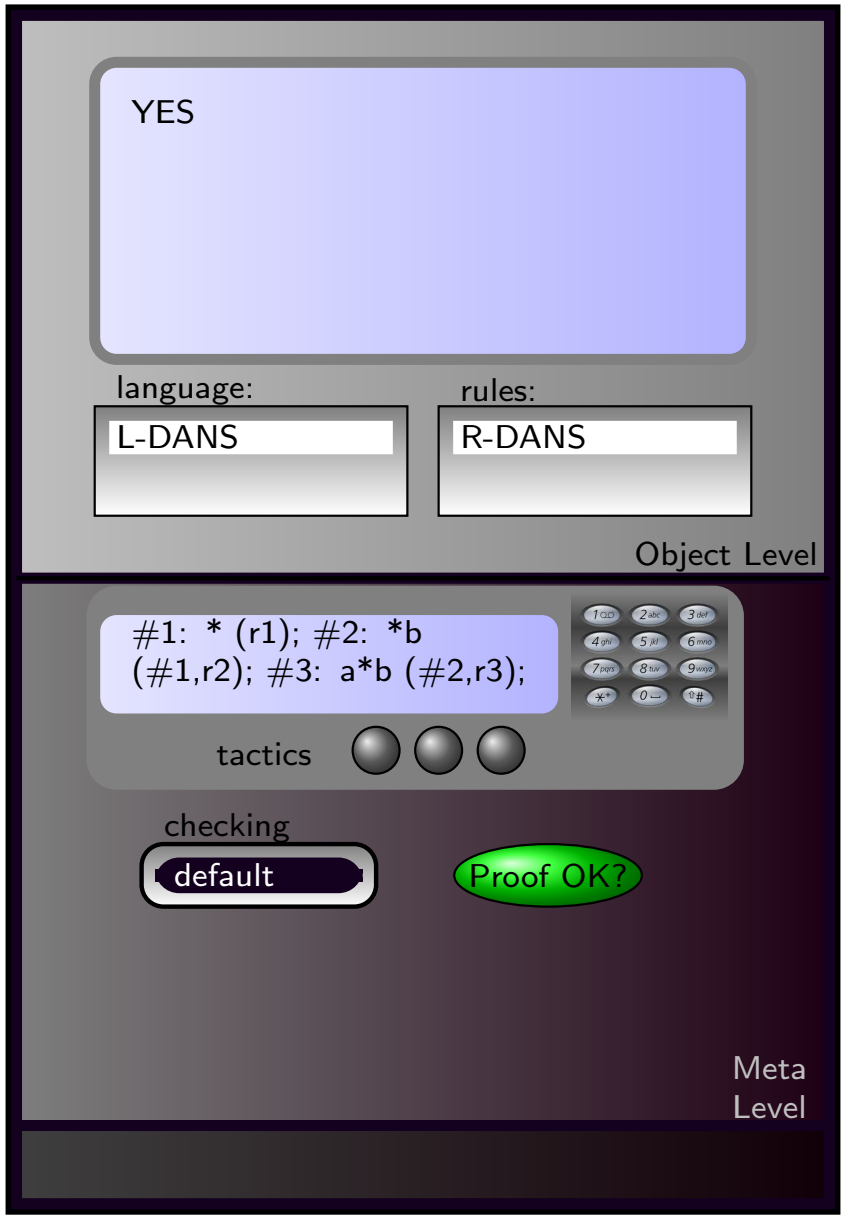

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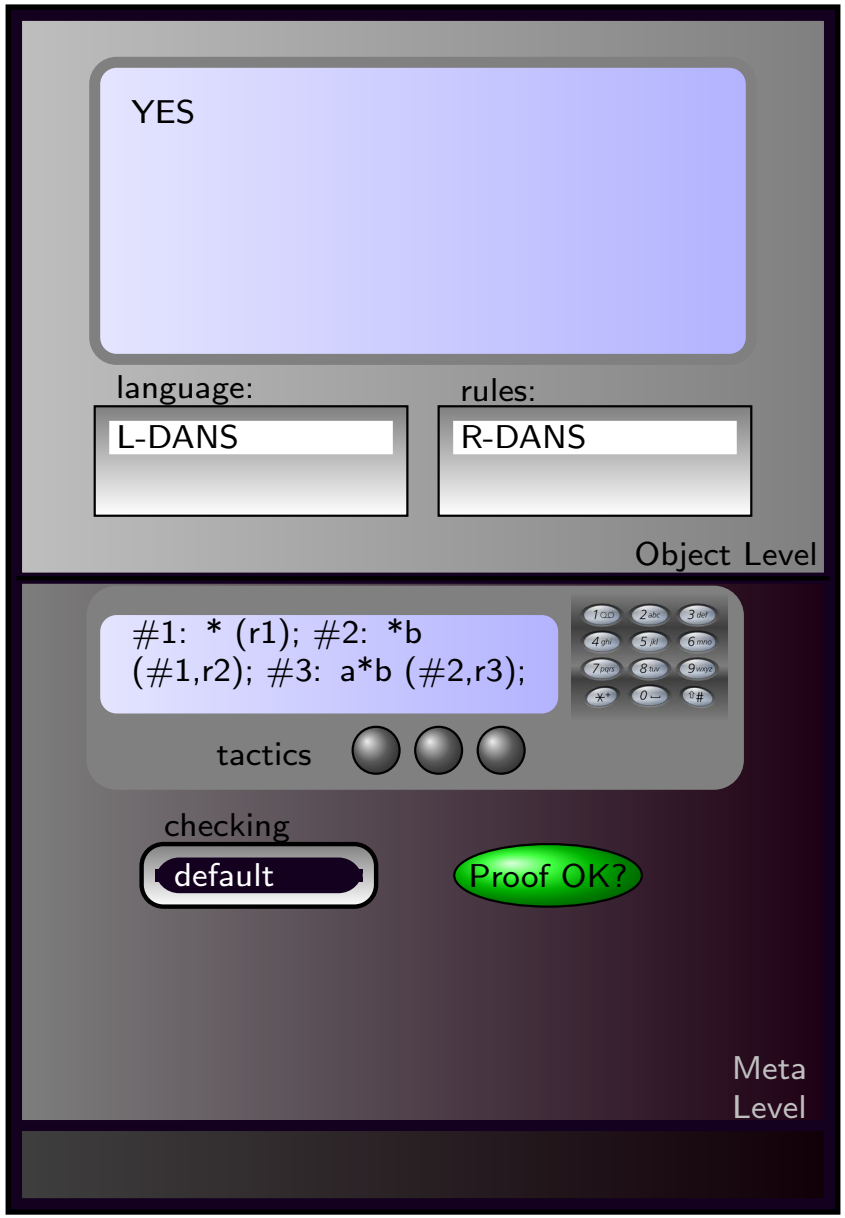

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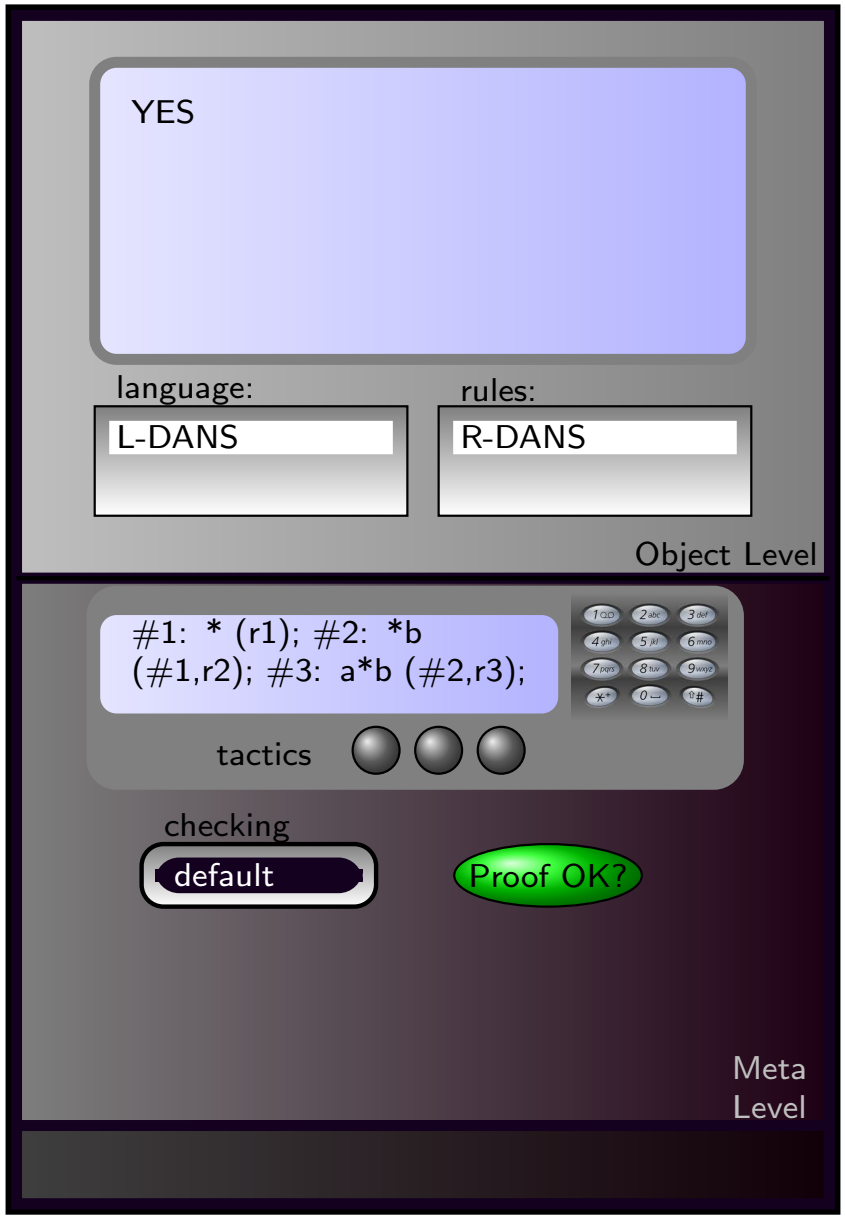

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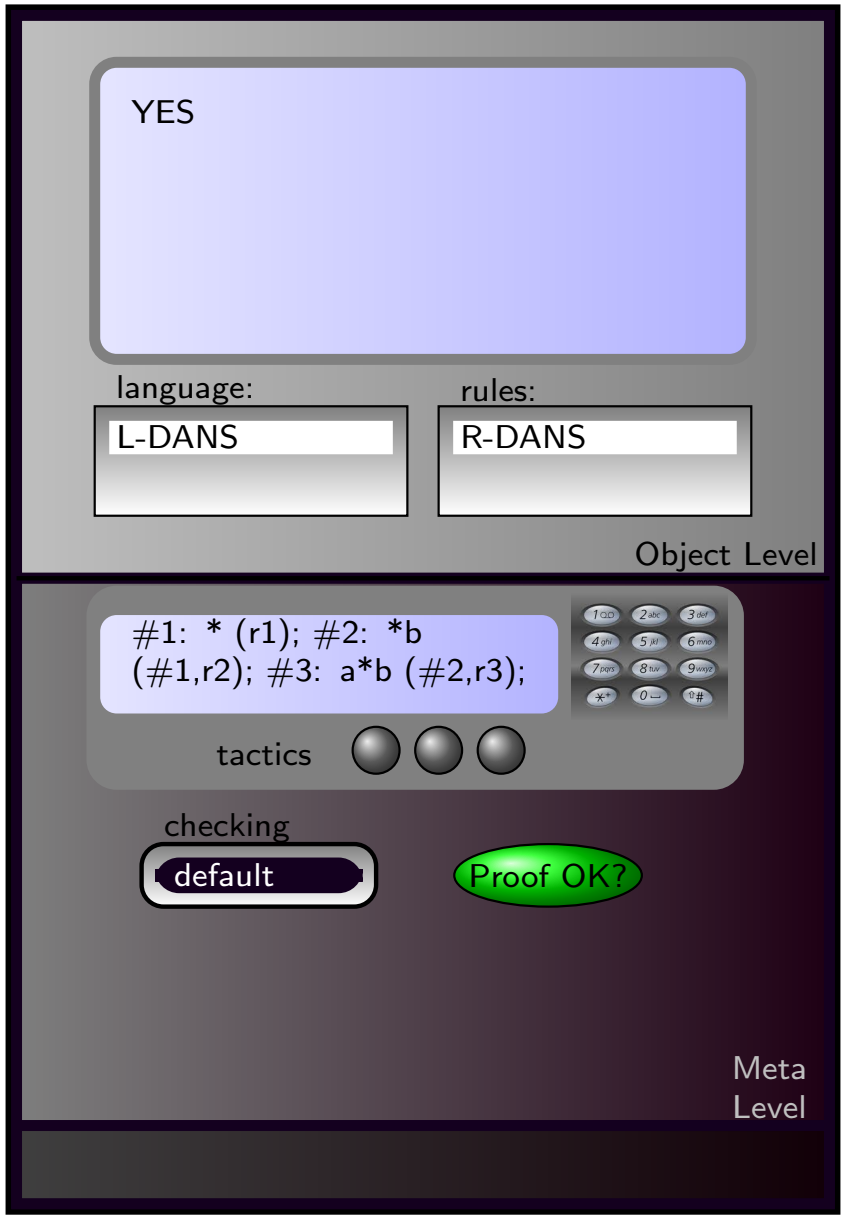

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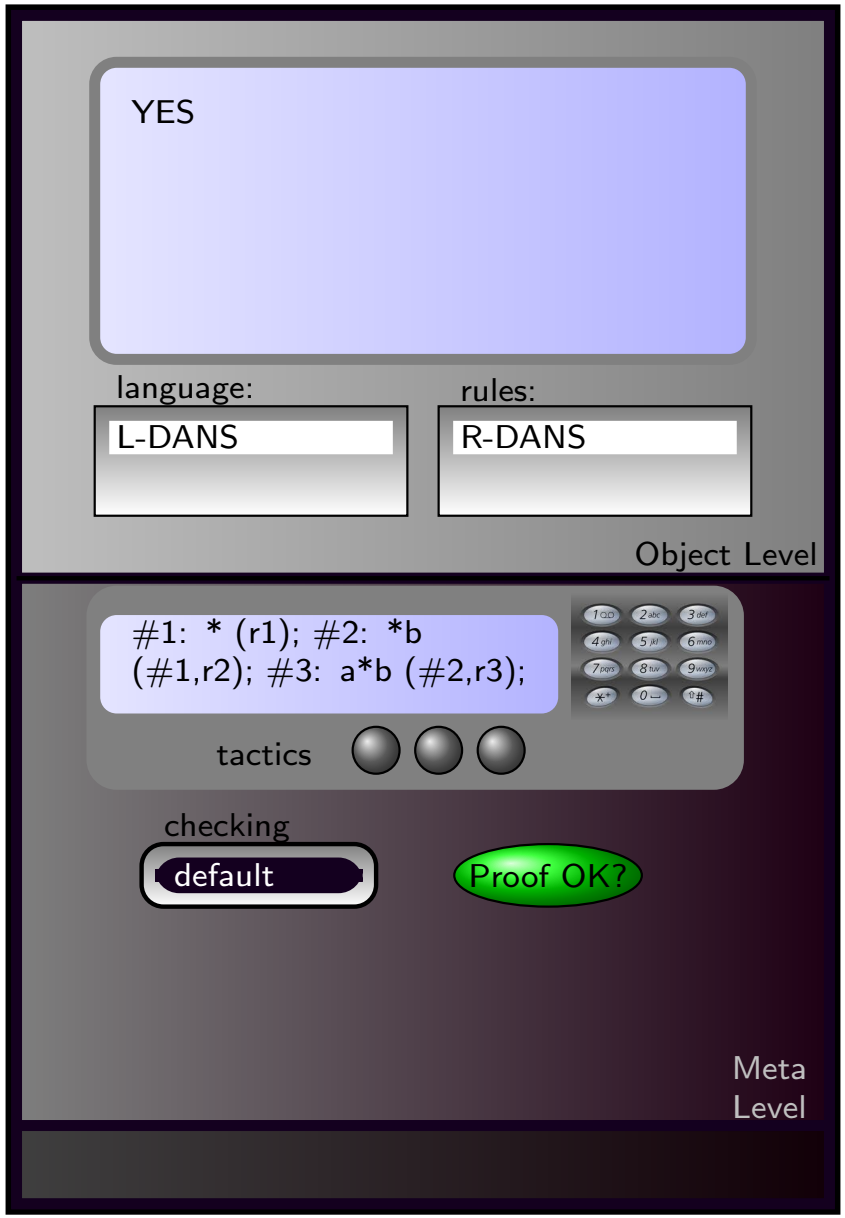
- We will use this facility when we study Coq and Isabelle in the 'Interactive Provers' course



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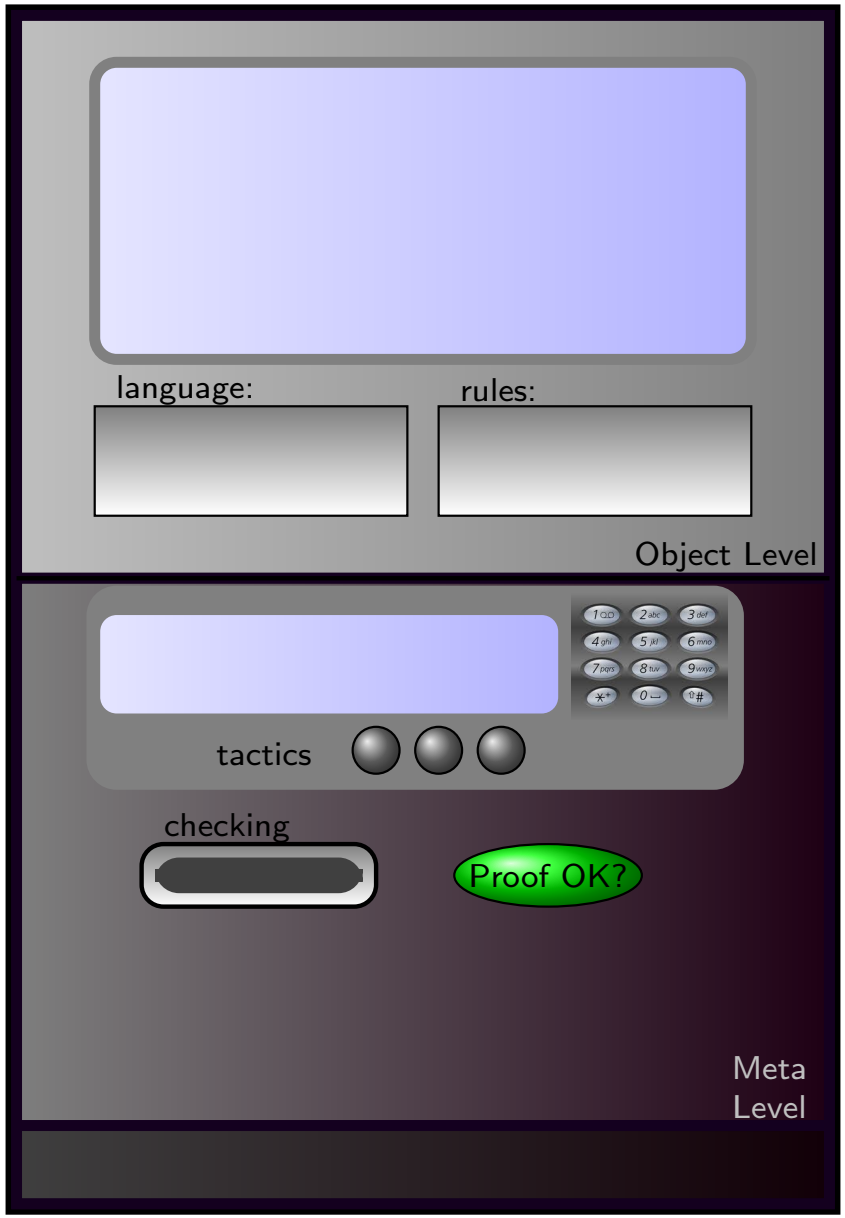

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- The default algorithm for checking whether a wff is a theorem is to let the machine run and record all theorems; if the wff is a theorem, eventually the machine will match it and output YES

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- We say that the system is *undecidable*; we saw that DANS is a decidable system, but DANS is simple

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  You will hopefully see through examples that undecidable type-checking in a stronger language is sometimes acceptable (you just press Ctrl-C to kill the compiler and rethink your code); it's better if the compiler hangs than if your code hangs in the field

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- The fourth rule says . . . well, she accepted the invitation

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- You will see that it is easier to cook up a system if you have an intended interpretation

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- Alternative: take DANS and make it more interesting, by adding more language and more rules

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- So we will sometimes look at a formal system as consisting of language and a decision procedure (without a proof calculus necessarily present)

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- The rules can be a proof calculus or a YES/NO decision procedure

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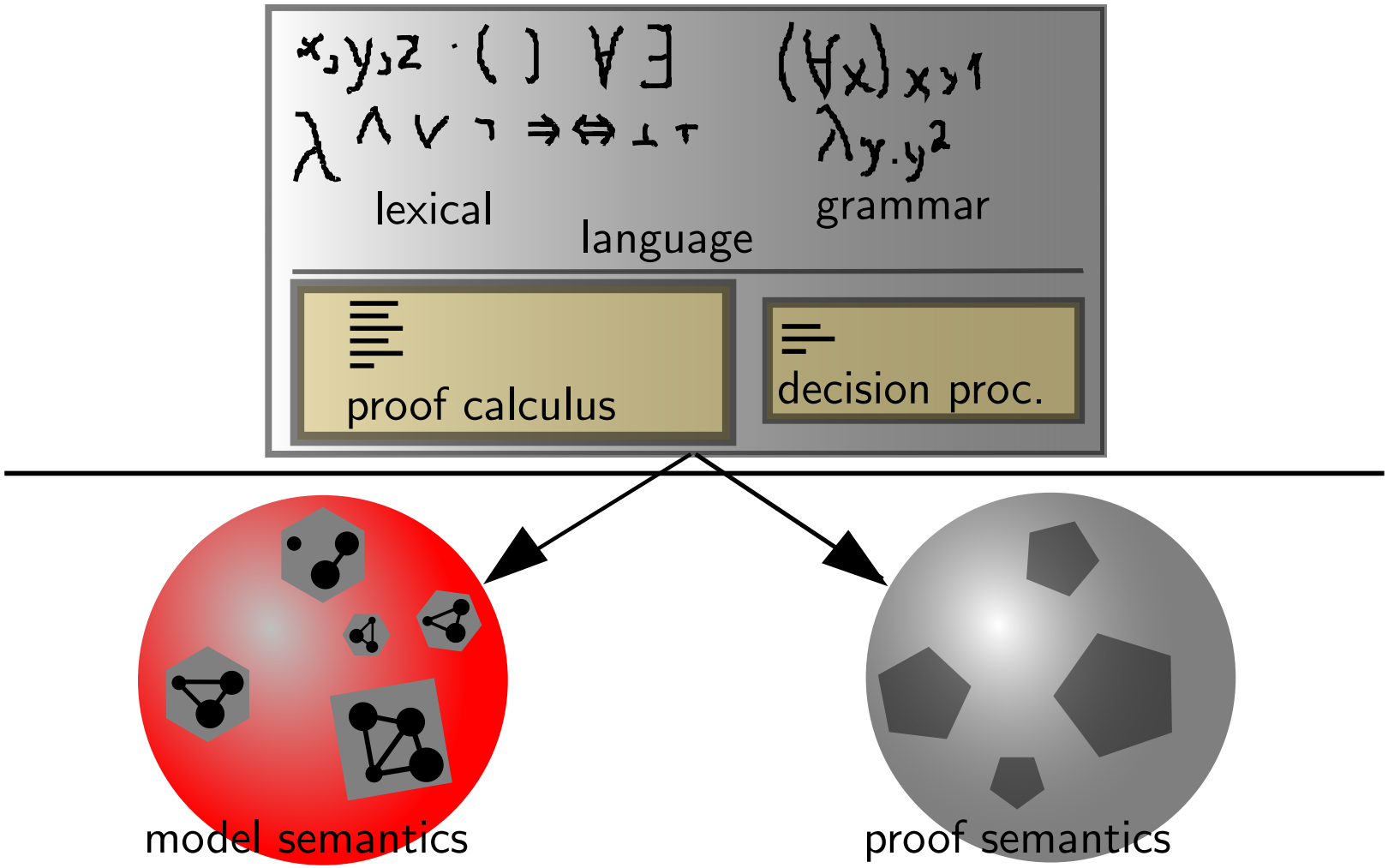
  - Model semantics (meaning of language is through the 'real' things that it represents)

Formal systems and their semantics



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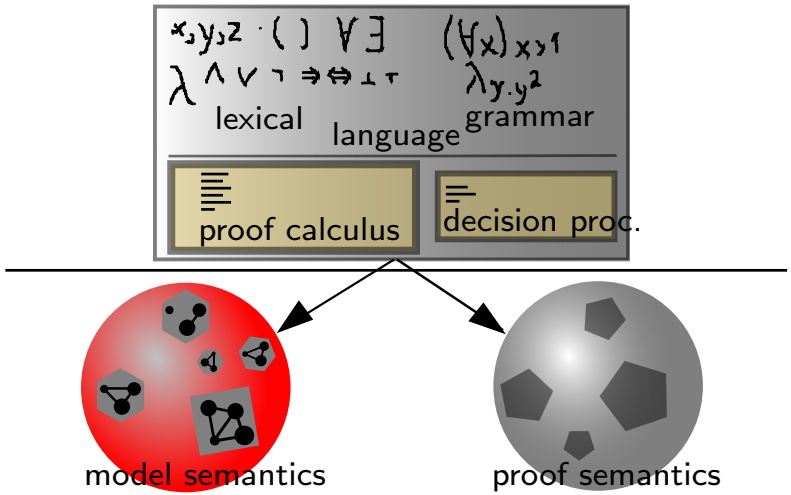
- Formal system at top, its semantics at bottom



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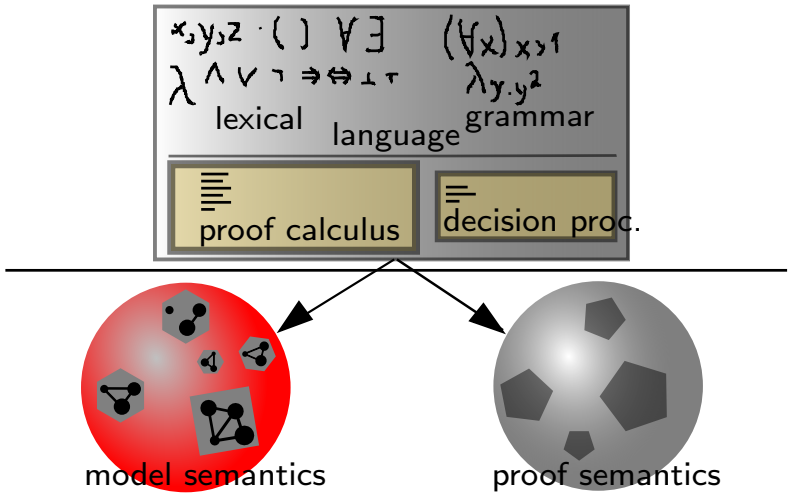
- Again, *formal system = language + proof calculus*



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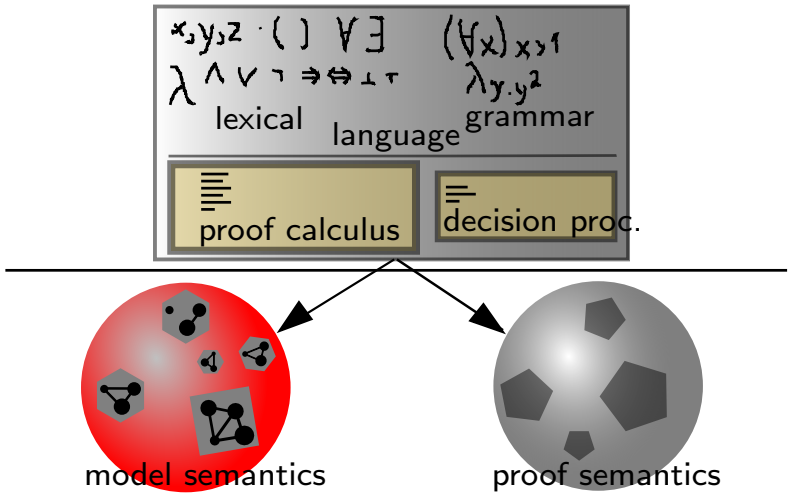

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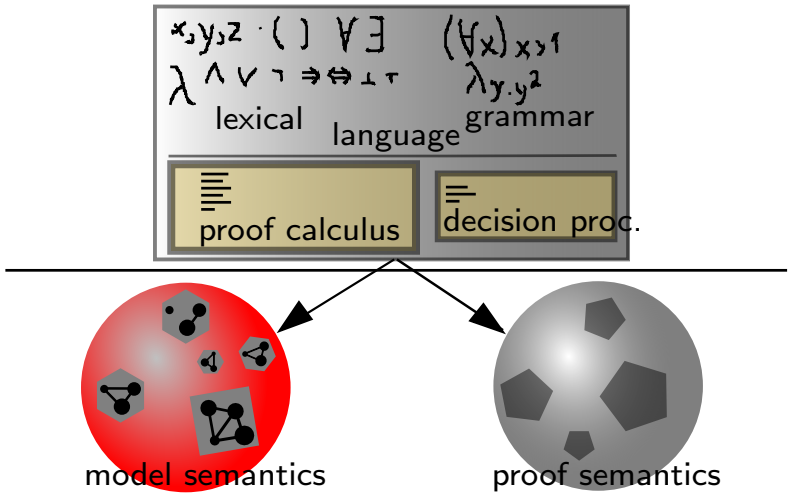

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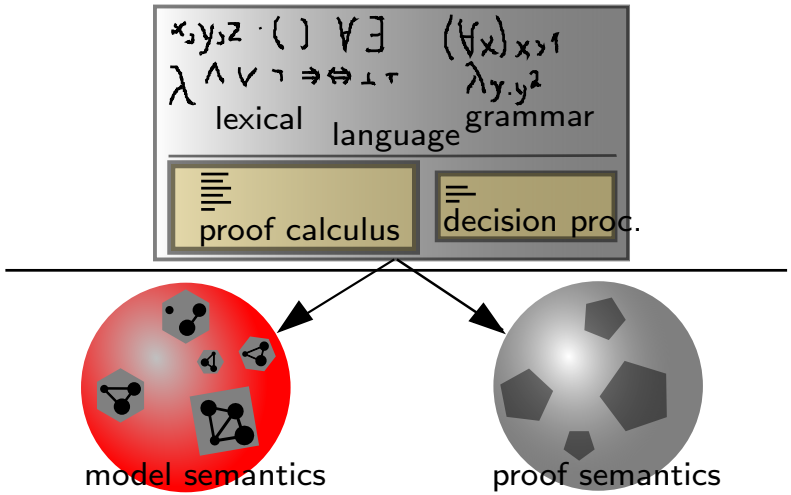

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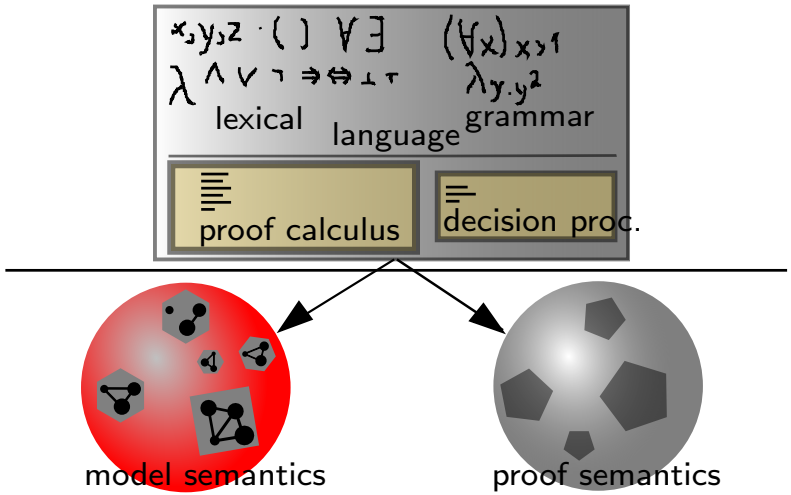

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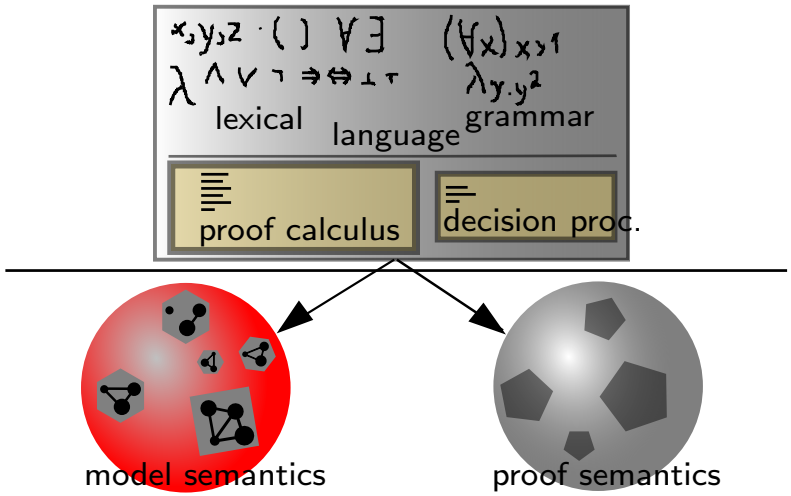

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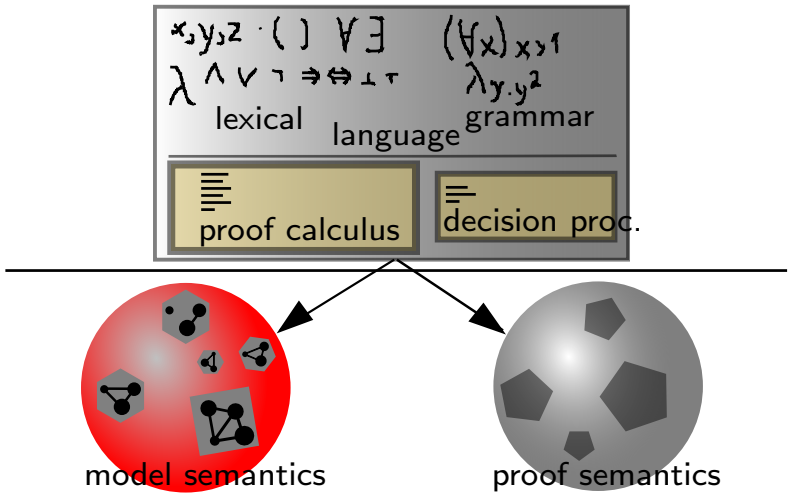

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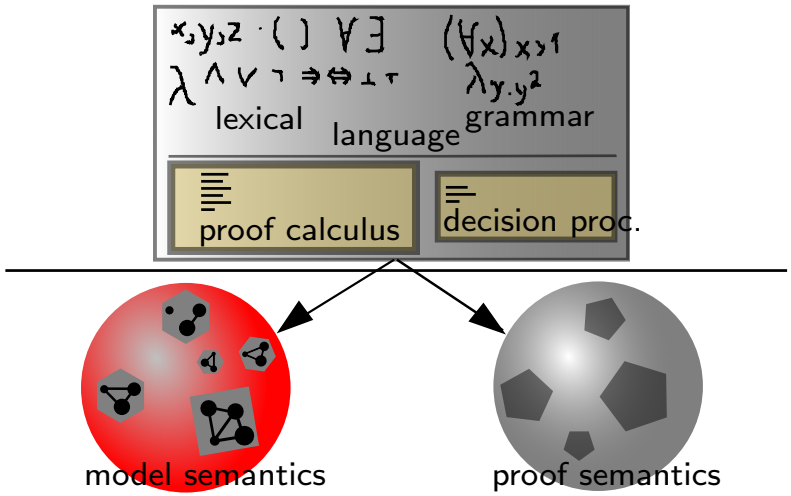

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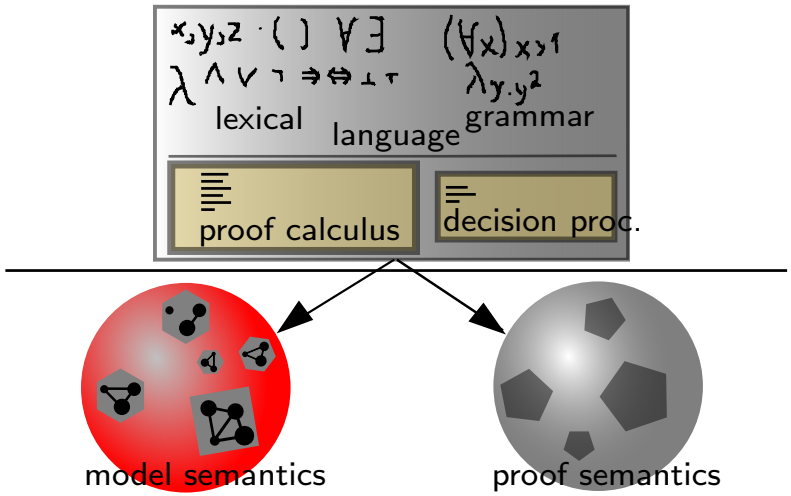

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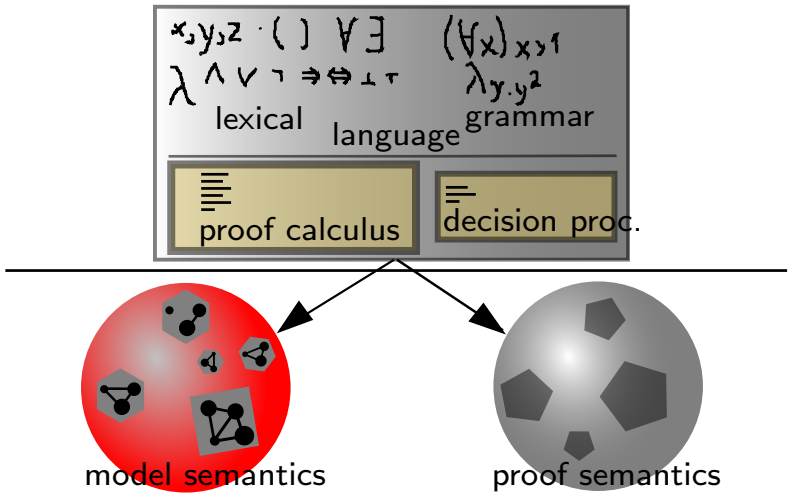
- This picture of the two basic semantics is good to keep in mind



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- We dedicate a course to it: 'Model Checking and Concurrency'

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- The system says that they *could*, but the girl can potentially refuse all night long, while more boys and girls swell up the lines

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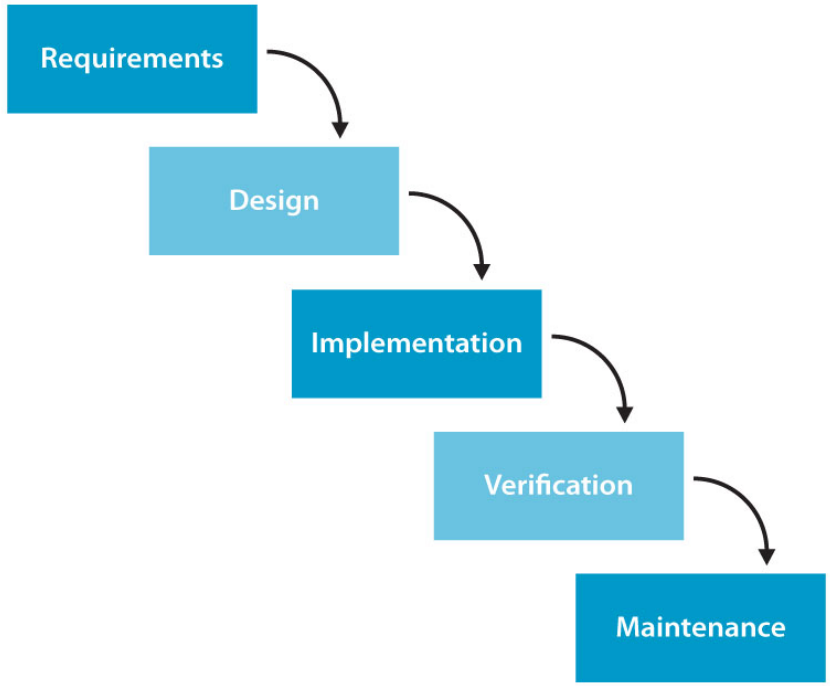
Proofs may be used during all phases of the software waterfall model:



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- The implementation and testing phases can also be treated formally, using a variety of other formal systems

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- Think about all the methodologies and the processes you learned in the past. *Have they helped with maintaining conceptual integrity?*

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- Because conceptual integrity correlates to a higher level of abstraction, which formalism also supports

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- Not a database of syntactic signatures, which can easily be (and often is) circumvented

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- So, why are formal methods so important in software engineering?
- The main reason is that all the tools used in software, from the computer, to operating system, to the compiler, to other applications, are essentially formal systems, i.e. specialized languages to be used according to set rules. So *it is the very essence of software that calls for the use of formal methods.*

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- Academia and industrial research have been interested for a long time, *but that's not sufficient*

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- More time should be spent with models than with programs

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- One of the largest machine-checked proofs ever done.

Remarkable success story: seL4

‘ Formal proofs for specific properties have been conducted for smaller kernels, but what we have done is a general, functional correctness proof which has never before been achieved for real-world, high-performance software of this complexity or size. ’

– Gerwin Klein, NICTA

‘ It is hard to comment on this achievement without resorting to clichés. Proving the correctness of 7,500 lines of C code in an operating system’s kernel is a unique achievement, which should eventually lead to software that meets currently unimaginable standards of reliability. ’

– Lawrence Paulson, Cambridge University Computer Laboratory

High complexity: the seL4 function call graph

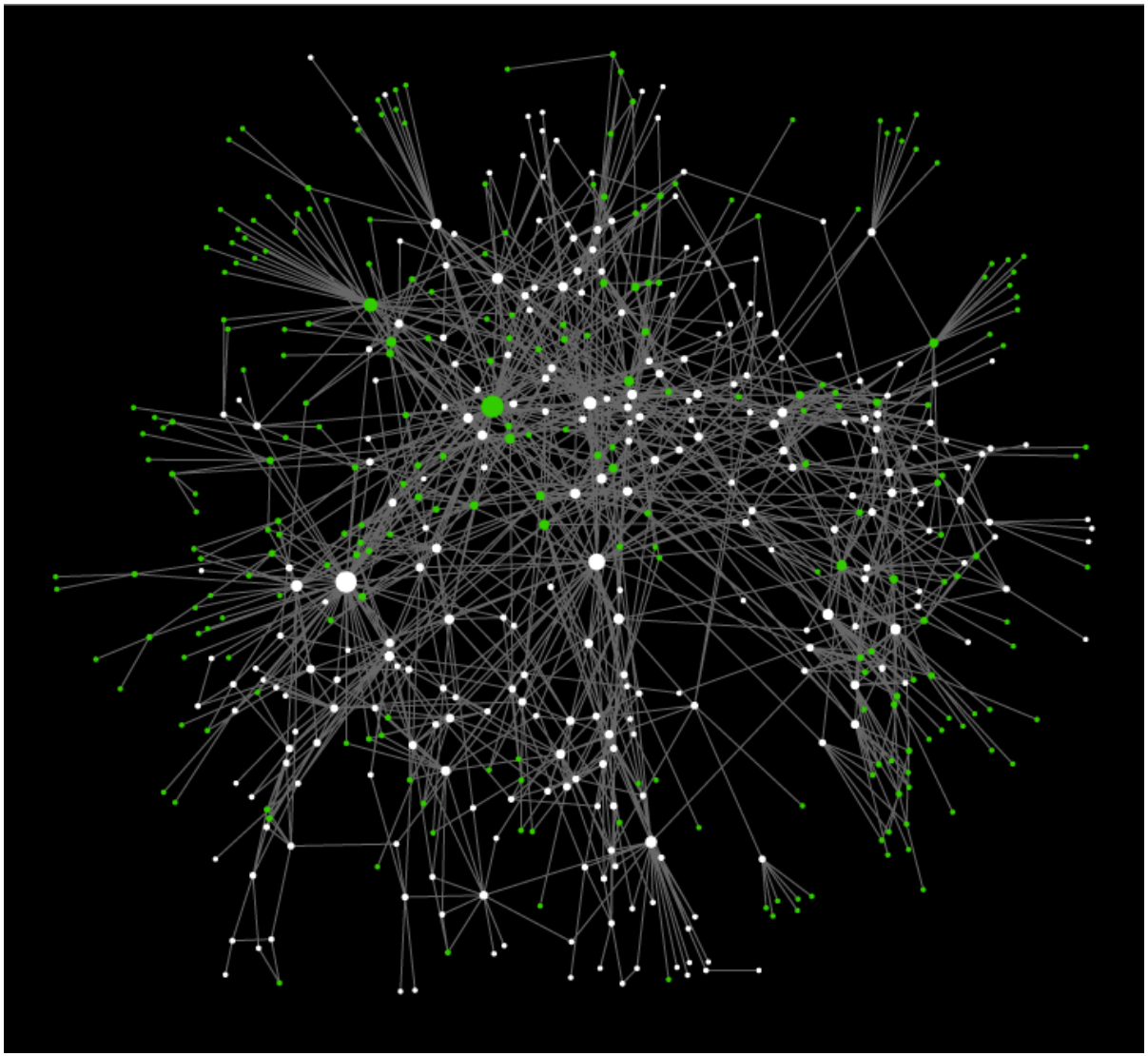
Vertices in the graph represent functions in the kernel

Edges between vertices are function calls

An abstract functional specification of seL4 is given

The C implementation is proven to satisfy this specification

http://ertos.nicta.com.au/research/



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- Formulate in your own words an explanation of what this might mean

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- If this example interests you, see `http://www.commoncriteriaportal.org/cc/`

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- We might get your organization interested in customizing/improving these (mostly open-source) tools

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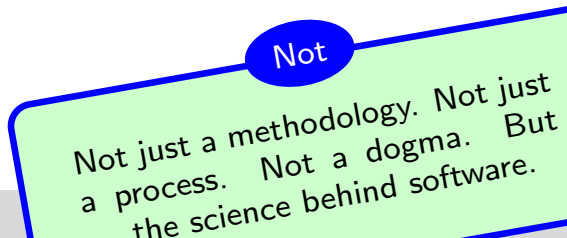

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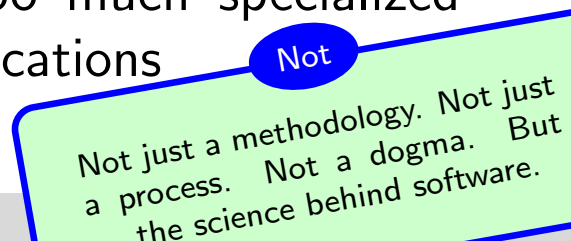

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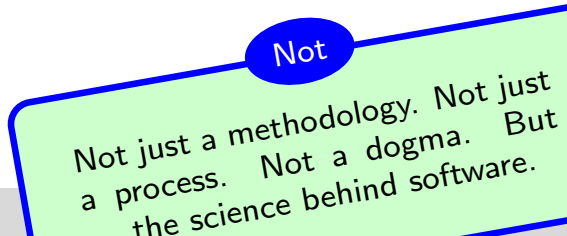

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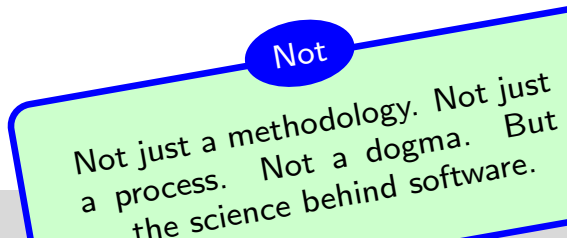

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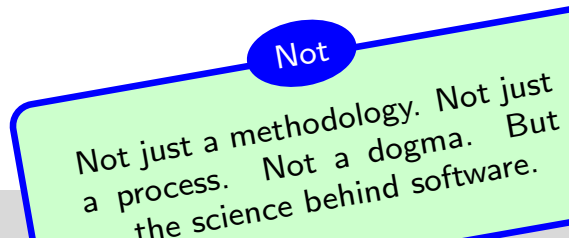

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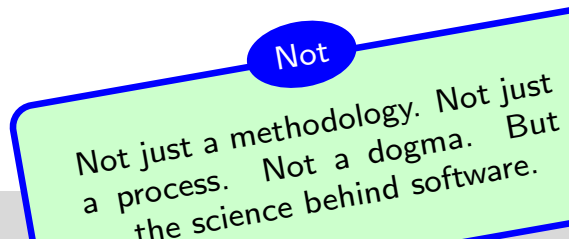

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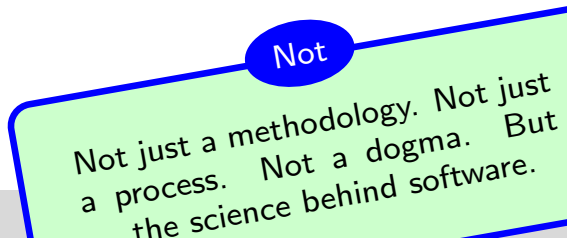
- *What is the science that software engineers need to know?*



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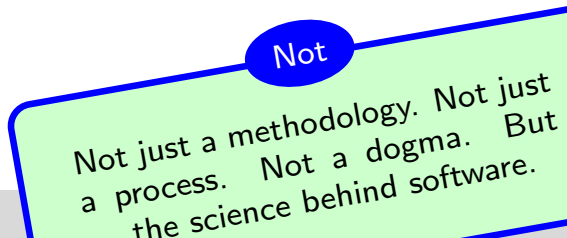

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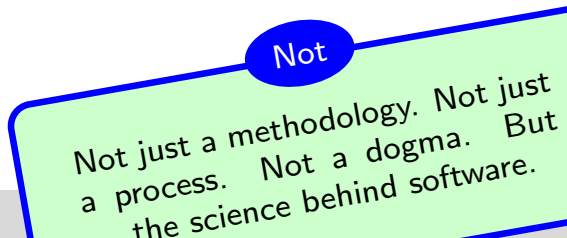
- We can prove ZFC consistency in a more powerful system than ZFC (which by the way is not finitary anymore), and then this system needs a proof of consistency in a still more powerful system, in a never ending chain.
- We cannot bootstrap this process.



If we cannot do that, what can we do?

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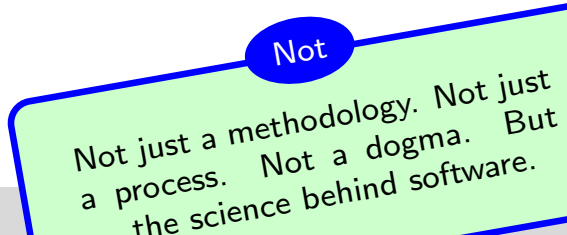

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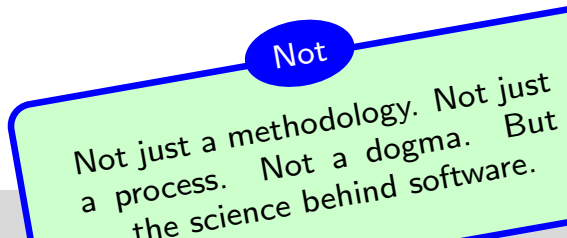

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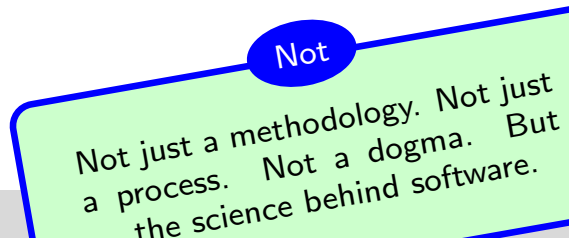

Relative correctness

The chain of trust

Assuming that:

- the hardware and the OS work properly
- the compiler of your logical tool works properly
- the logic behind the tool is consistent
- the tool implements the logic correctly
- your theory is consistent with the logic
- your proof is checked by the logical tool

then

- your proof is correct.

Not

Not just a methodology. Not just a process. Not a dogma. But the science behind software.

Strongest claim we can make

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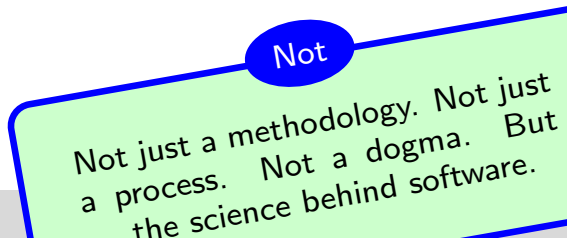

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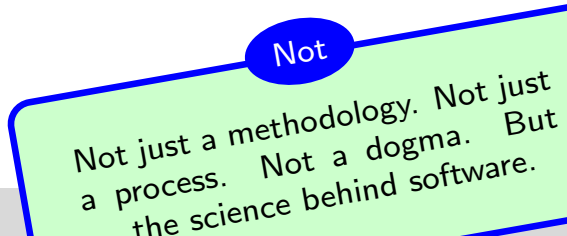

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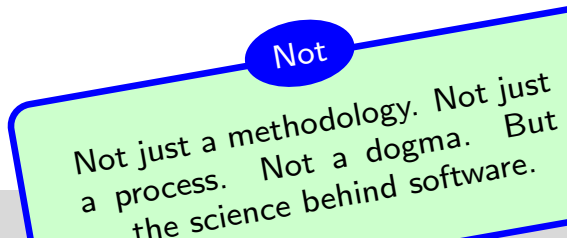

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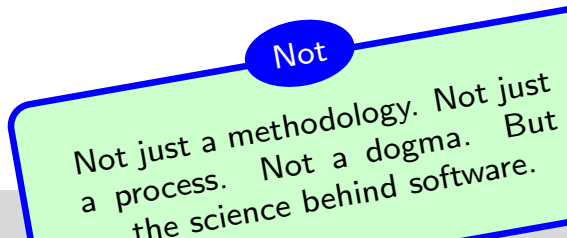

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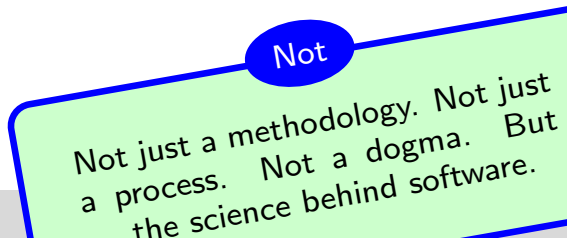

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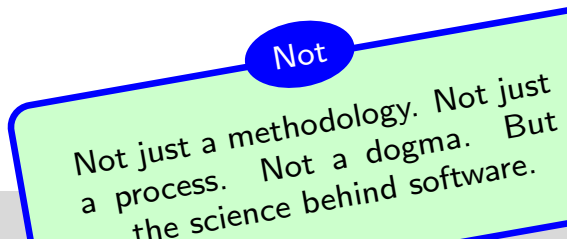
- The correctness of a tool's implementation is a subject of much research



Not just another methodology or process

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Not just a methodology. Not just a process. Not a dogma. But the science behind software.

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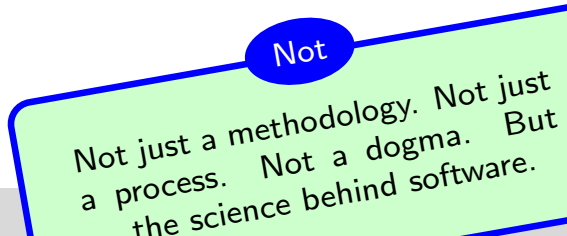
- We have had many methodologies: structured, object-oriented, extreme, test-driven, ...



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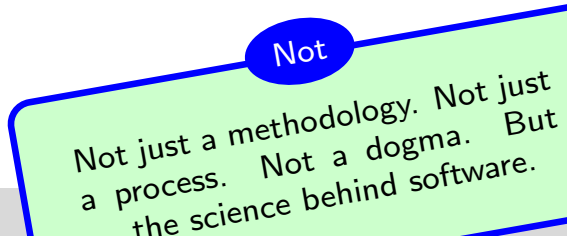

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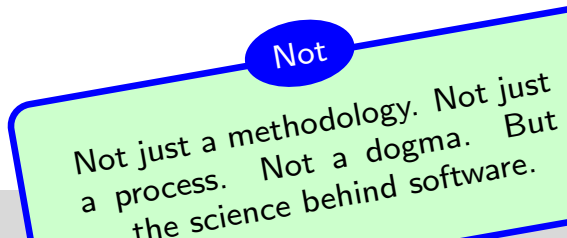

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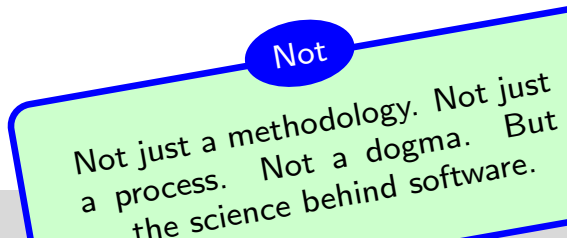

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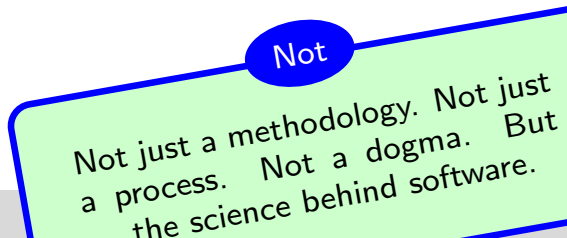

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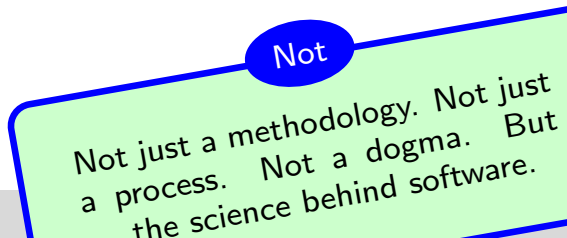
- We will actually spend some time looking at formal methodology and process
- But the main point is that formal systems are the scientific basis of software



Are there two kinds of software?

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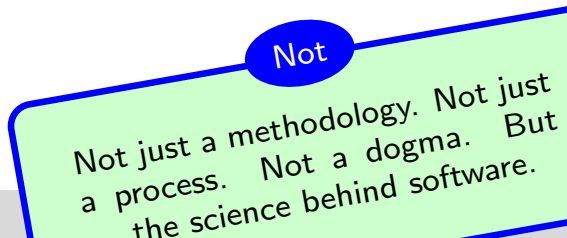
Are there two kinds of software?

- It is said that formal methods are too expensive



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It this true?

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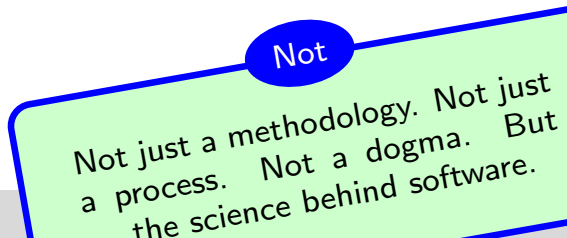
It this true? Of course it is true, and if we base our value judgements on testing, software will always fall into the second category, regardless of when the testing is done or how good it is. Quite often, decision makers are not even aware that there is an alternative.



The hype issue

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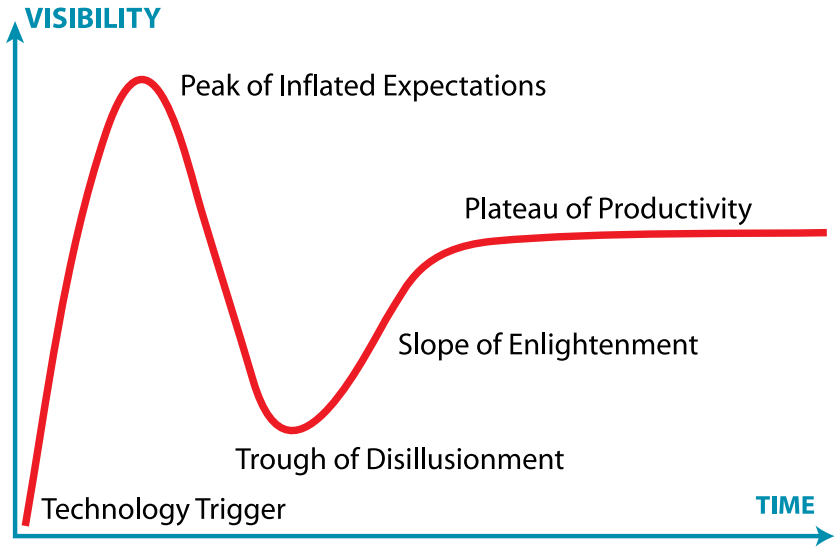

On Wikipedia, the 'hype curve' is defined as a 'graphic representation of the maturity, adoption and social application of specific technologies'. It matches quite well with the trajectory of many technologies that were used for software development.

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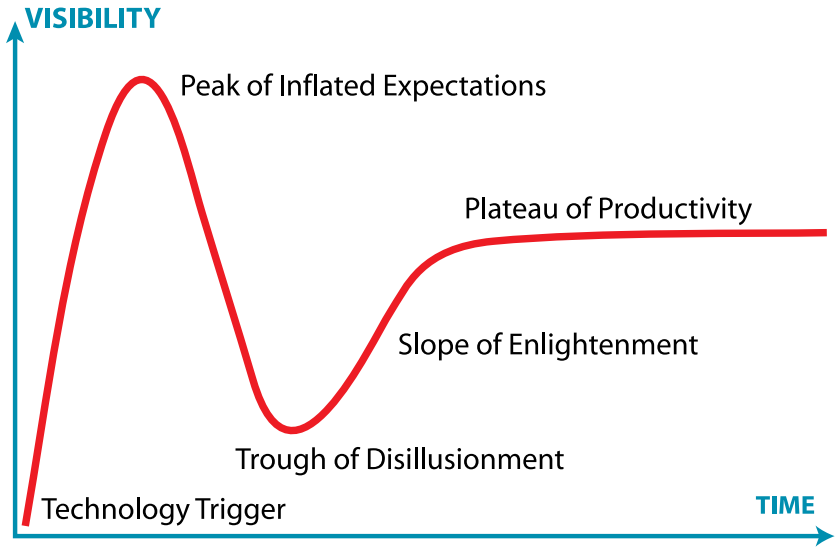

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It has also been mentioned in the context of formal methods. *But formal methods are not just a technology. Their scientific component has continuously advanced upward, as any science does.*

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Not just software with math but also math with software

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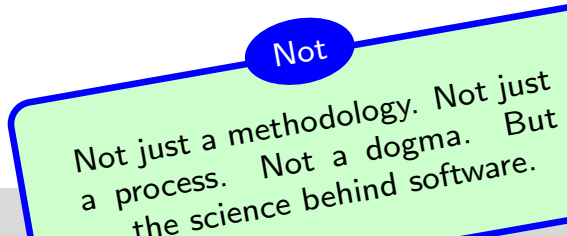
- Our definition shows a one-way dependency of formal software development on mathematics



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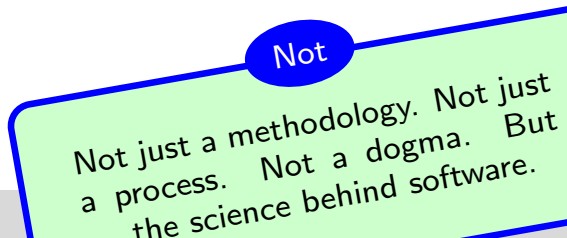

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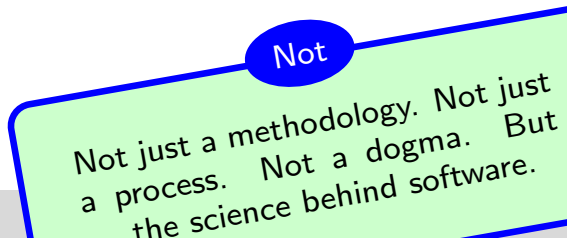

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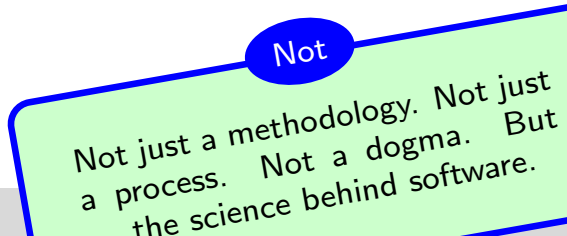

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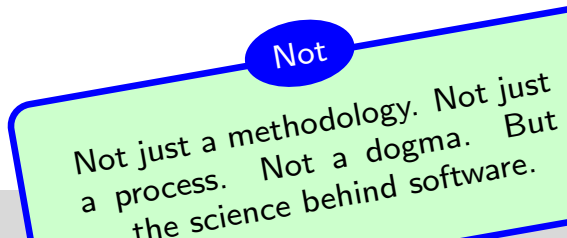

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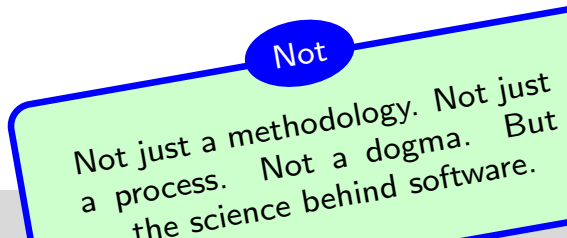
  - The extreme cases that are behind those limiting results of mathematics rarely pop up in practical situations (the kind we encounter in software development)



Proofs do not replace engineering

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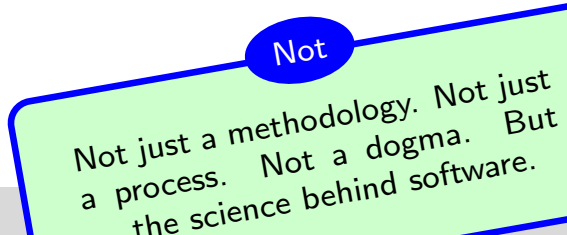

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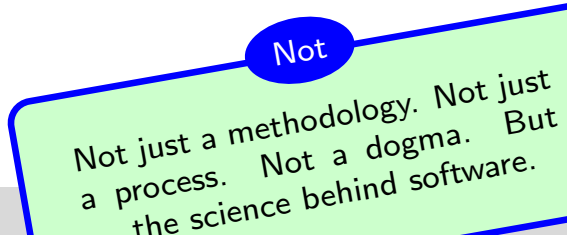

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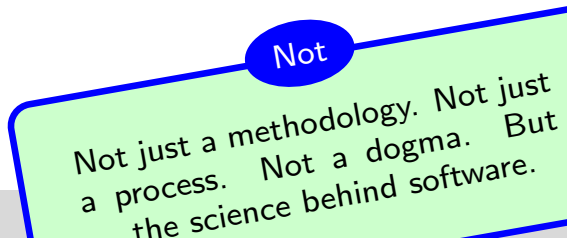

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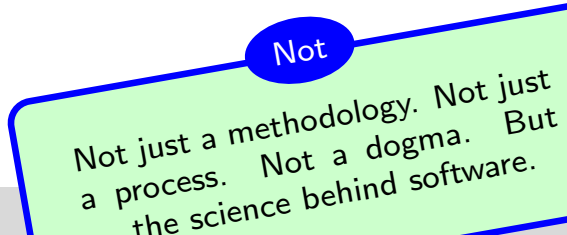
- That issue is important though, and *only our engineering expertize can address it*



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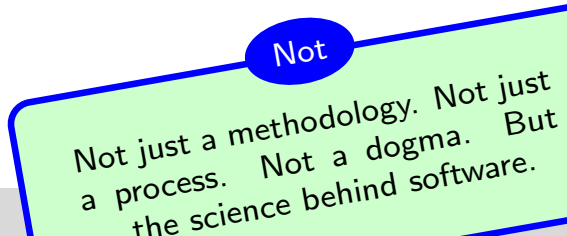

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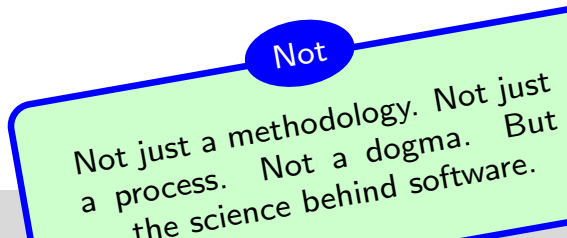

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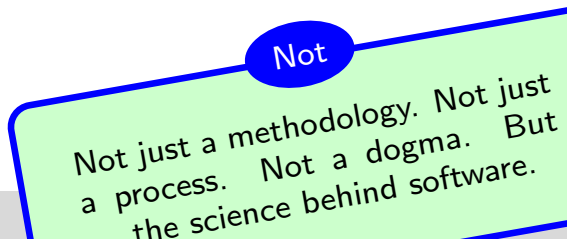

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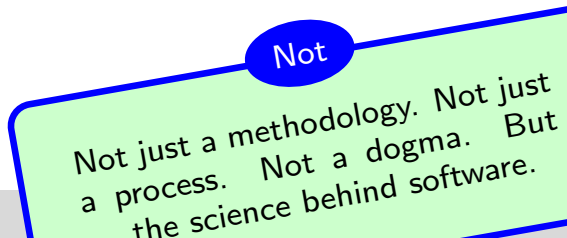

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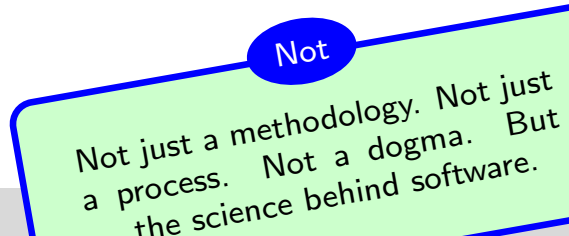

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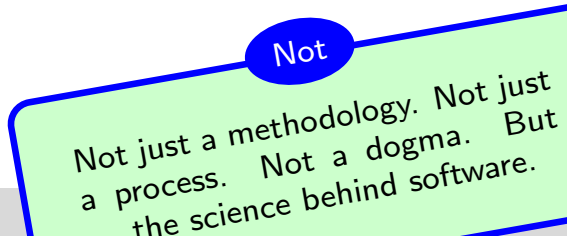

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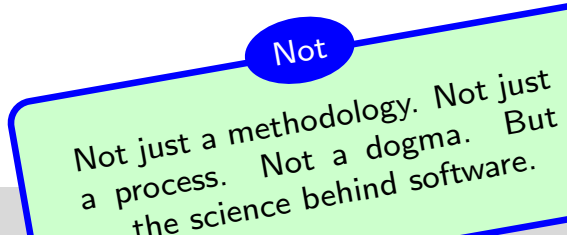

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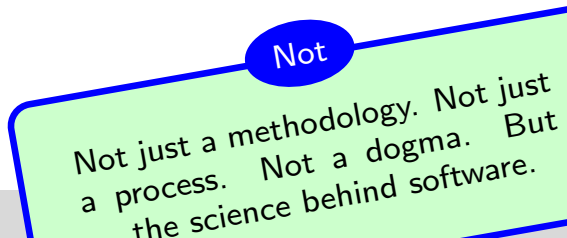

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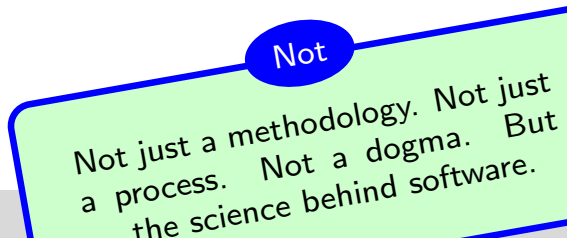

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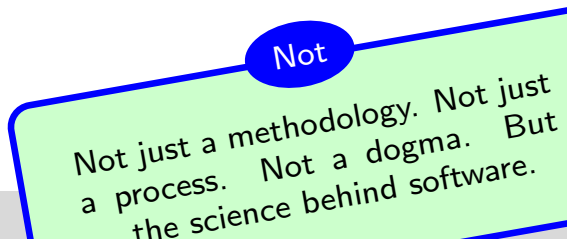

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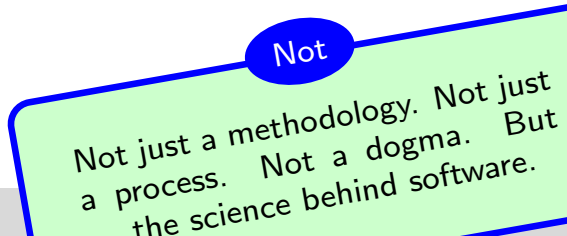

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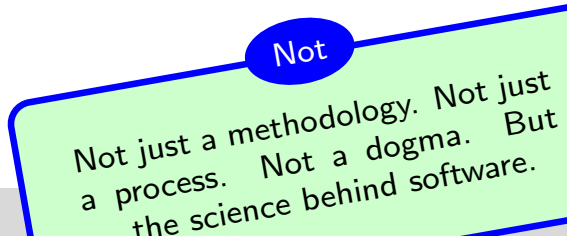

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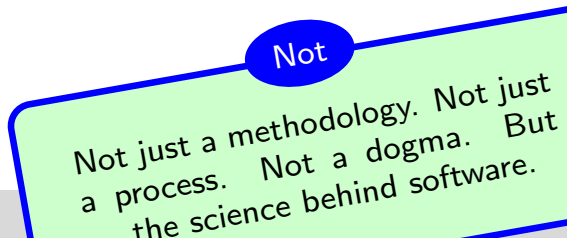

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- So, although the topic of this section is formal program verification, we always *keep the entire formal development cycle in mind*

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- We begin by looking at functional programs first

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  4. Using monads (another reason for us to study category theory, this being the more general approach; the other methods can be described through monads)

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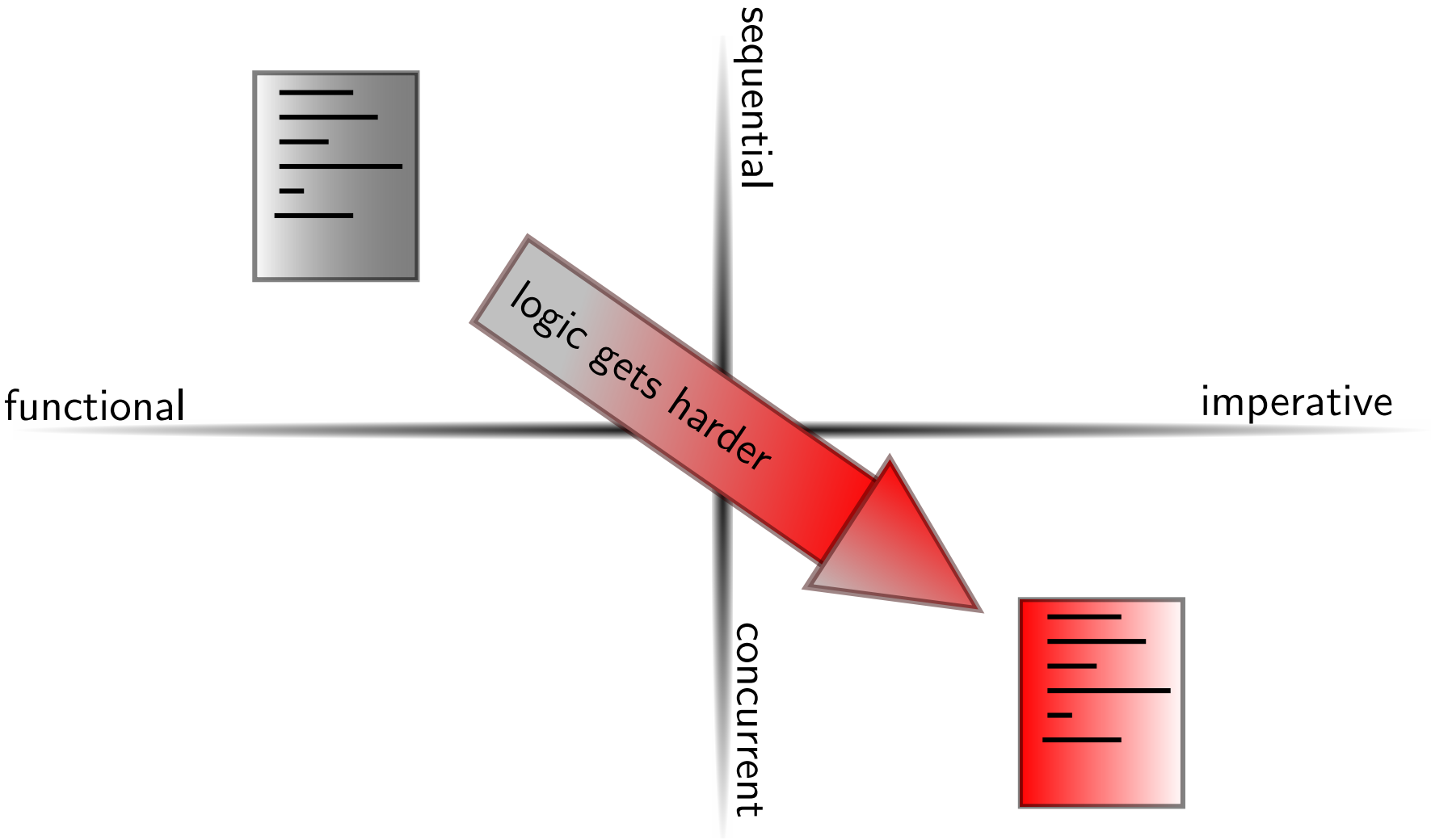
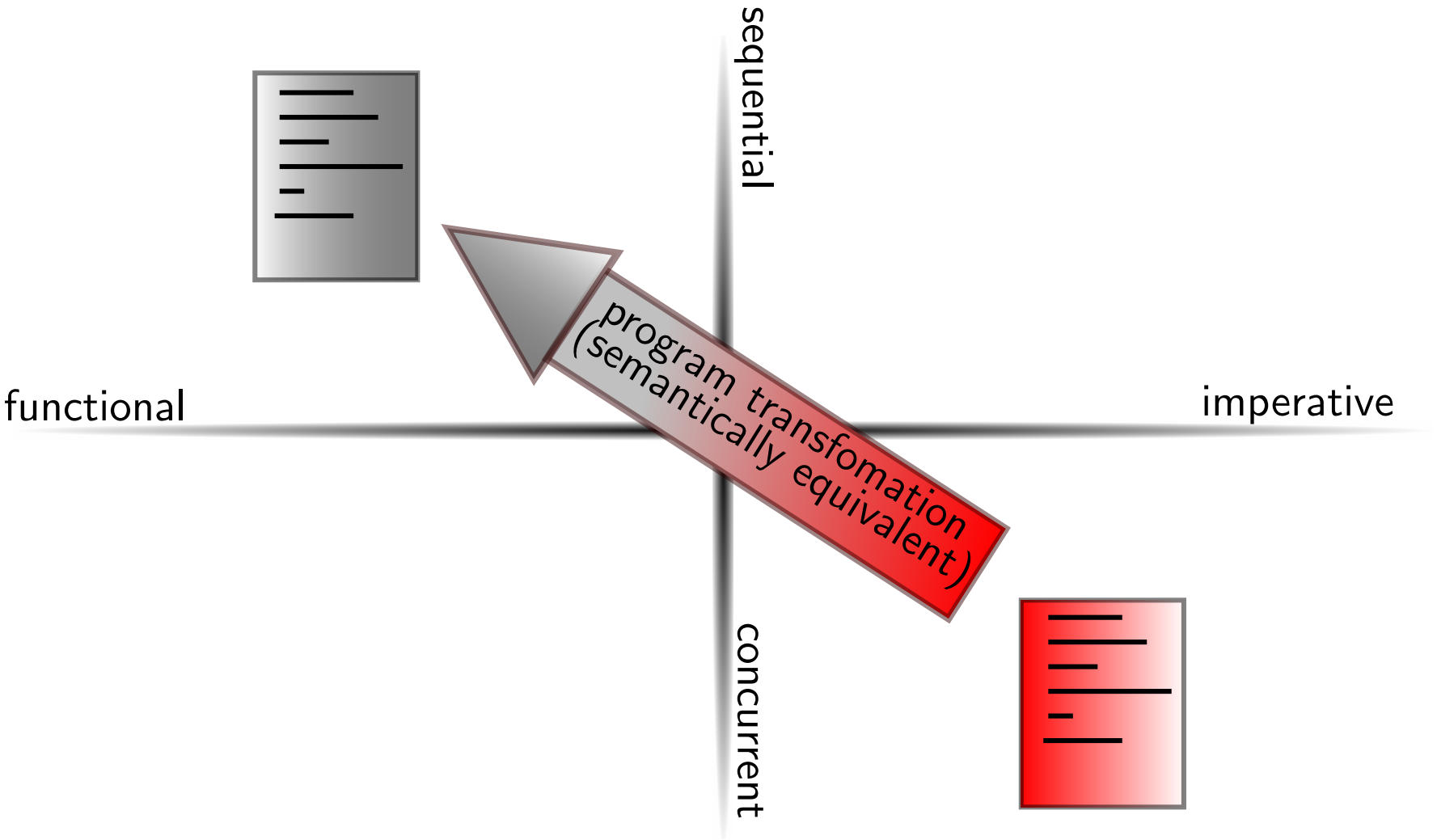
- Gödel himself was not convinced that his theory of recursive functions or Church's lambda calculus captured the concept of computability until Turing proposed his machine; imperative models are usually more convincing.

Impact of concurrency



Solution: program transformations



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- *Termination* (results *are* delivered)

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- *Liveness* (program *does not* terminate)

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- *Fairness* (needed to ensure that all components that are ready for execution do indeed get a chance to execute)

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- We use debug assertions to validate our understanding of what the running state should be at some control point
- We set debugger breakpoints at control points to confirm a certain scenario. These scenarios are in fact implicit assertions, many times including logical connectives, assertions which the debugger may or may not confirm; the scenarios are mostly based on informal API documentation.

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- The verifying compiler (a smarter compiler, backed by a logical tool) combines these assertions with the formal semantics of the programming language being used and obtains formulas (i.e. theorems) which its logical back-end must subsequently prove.

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One of the most intuitive and most used formal semantics for imperative programming languages. Your program s is a composition of various commands written in the language. Then its behavior can be viewed as a state (or predicate) transformer:

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- Then use *induction on the structure of the program* s to prove that if the state of the system satisfies P and s is executed, then upon termination of s, the state of the system satisfies Q. This is partial correctness, because we still need to separately prove that s does indeed terminate.

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- But assertions could be attached *while writing the code*: JML, Spec#

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- So assertions force the writing of better code

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3. Assertions are the best comments, they represent the *logic* behind the program

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This is clearly an important issue since it may very well be that your code calls more operating-system or platform (Java/.NET) API functions than functions that are defined in your program (and to which you can add as many assertions as you need). These user-mode APIs have complicated semantics. So, are such pre-conditions, post-conditions and invariants being developed for operating-system or platform APIs?

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2. Yes (but not enough), we'll look at some solutions for Windows and .NET

User-mode APIs have complicated semantics

UNIX/LINUX EXAMPLE:

(This example uses traditional System V semaphores, a Posix semaphore would have a similarly complicated semantics.)

NAME int semget(key_t key, int nsems, int semflg);

DESCRIPTION This function returns the semaphore set identifier associated with the argument key. A new set of nsems semaphores is created if key has the value IPC_PRIVATE or if no existing semaphore set is associated to key and IPC_CREAT is asserted in semflg (i.e. semflg & IPC_CREAT isn't zero). The presence in semflg of the fields IPC_CREAT and IPC_EXCL plays the same role, with respect to the existence of the semaphore set, as the presence of O_CREAT and O_EXCL in the mode argument of the open(2) system call: i.e. the semget function fails if semflg asserts both IPC_CREAT and IPC_EXCL and a semaphore set already exists for key. Upon creation, the low-order 9 bits of the argument semflg define the access permissions (for owner, group and others) for the semaphore set. These bits have the same format, and the same meaning, as the mode argument in the open(2) or creat(2) system calls (though the execute permissions are not meaningful for semaphores, and write permissions mean permission to alter semaphore values).

When creating a new semaphore set, semget initializes the semaphore set's associated data structure semid_ds as follows:

sem_perm.cuid and sem_perm.uid are set to the effective user-ID of the calling process. sem_perm.cgid and sem_perm.gid are set to the effective group-ID of the calling process. The low-order 9 bits of sem_perm.mode are set to the low-order 9 bits of semflg. sem_nsems is set to the value of nsems. sem_otime is set to 0. sem_ctime is set to the current time. The argument nsems can be 0 (a don't care) when a semaphore set is not being created. Otherwise nsems must be greater than 0 and less than or equal to the maximum number of semaphores per semaphore set (SEMMSL).

If the semaphore set already exists, the access permissions are verified.

User-mode APIs have complicated semantics

RETURN VALUE If successful, the return value will be the semaphore set identifier (a nonnegative integer), otherwise -1 is returned, with errno indicating the error. ERRORS On failure errno will be set to one of the following: EACCES A semaphore set exists for key, but the calling process does not have permission to access the set. EEXIST A semaphore set exists for key and semflg was asserting both IPC_CREAT and IPC_EXCL. ENOENT No semaphore set exists for key and semflg wasn't asserting IPC_CREAT. EINVAL nsems is less than 0 or greater than the limit on the number of semaphores per semaphore set (SEMMSL), or a semaphore set corresponding to key already exists, and nsems is larger than the number of semaphores in that set. ENOMEM A semaphore set has to be created but the system has not enough memory for the new data structure. ENOSPC A semaphore set has to be created but the system limit for the maximum number of semaphore sets (SEMMNI), or the system wide maximum number of semaphores (SEMMNS), would be exceeded.

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Windows example:

Syntax HANDLE WINAPI CreateSemaphore(__in_opt LPSECURITY_ATTRIBUTES lpSemaphoreAttributes, __in LONG lInitialCount, __in LONG lMaximumCount, __in_opt LPCTSTR lpName);

Parameters lpSemaphoreAttributes [in, optional] A pointer to a SECURITY_ATTRIBUTES structure. If this parameter is NULL, the handle cannot be inherited by child processes.

The lpSecurityDescriptor member of the structure specifies a security descriptor for the new semaphore. If this parameter is NULL, the semaphore gets a default security descriptor. The ACLs in the default security descriptor for a semaphore come from the primary or impersonation token of the creator.

lInitialCount [in] The initial count for the semaphore object. This value must be greater than or equal to zero and less than or equal to lMaximumCount. The state of a semaphore is signaled when its count is greater than zero and nonsignaled when it is zero. The count is decreased by one whenever a wait function releases a thread that was waiting for the semaphore. The count is increased by a specified amount by calling the ReleaseSemaphore function.

lMaximumCount [in] The maximum count for the semaphore object. This value must be greater than zero.

lpName [in, optional] The name of the semaphore object. The name is limited to MAX_PATH characters. Name comparison is case sensitive.

If lpName matches the name of an existing named semaphore object, this function requests the SEMAPHORE_ALL_ACCESS access right. In this case, the lInitialCount and lMaximum-Count parameters are ignored because they have already been set by the creating process. If the lpSemaphoreAttributes parameter is not NULL, it determines whether the handle can be inherited, but its security-descriptor member is ignored.

If lpName is NULL, the semaphore object is created without a name.

...

User-mode APIs have complicated semantics

Return Value If the function succeeds, the return value is a handle to the semaphore object. If the named semaphore object existed before the function call, the function returns a handle to the existing object and GetLastError returns ERROR_ALREADY_EXISTS.

If the function fails, the return value is NULL. To get extended error information, call GetLastError.

Remarks The handle returned by CreateSemaphore has the SEMAPHORE_ALL_ACCESS access right; it can be used in any function that requires a handle to a semaphore object, provided that the caller has been granted access. If an semaphore is created from a service or a thread that is impersonating a different user, you can either apply a security descriptor to the semaphore when you create it, or change the default security descriptor for the creating process by changing its default DACL. For more information, see Synchronization Object Security and Access Rights.

Any thread of the calling process can specify the semaphore-object handle in a call to one of the wait functions. The single-object wait functions return when the state of the specified object is signaled. The multiple-object wait functions can be instructed to return either when any one or when all of the specified objects are signaled. When a wait function returns, the waiting thread is released to continue its execution.

The state of a semaphore object is signaled when its count is greater than zero, and nonsignaled when its count is equal to zero. The lInitialCount parameter specifies the initial count. Each time a waiting thread is released because of the semaphore's signaled state, the count of the semaphore is decreased by one. Use the ReleaseSemaphore function to increment a semaphore's count by a specified amount. The count can never be less than zero or greater than the value specified in the lMaximumCount parameter.

Multiple processes can have handles of the same semaphore object, enabling use of the object for interprocess synchronization. The following object-sharing mechanisms are available:

A child process created by the CreateProcess function can inherit a handle to a semaphore object if the lpSemaphoreAttributes parameter of CreateSemaphore enabled inheritance. A process can specify the semaphore-object handle in a call to the DuplicateHandle function to create a duplicate handle that can be used by another process.

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3. we'll learn in the program the VCC tool used for the verification of Hyper-V

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- Our program is obviously *more focussed on theorem proving and model checking than static analysis*

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- There is a large step when moving from static code analysis to theorem proving/model checking

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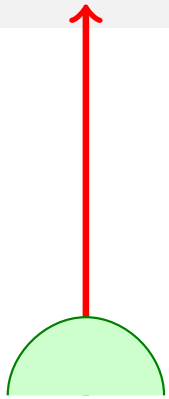

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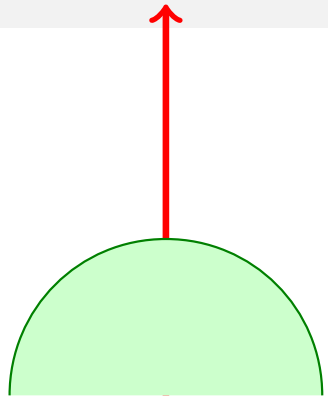

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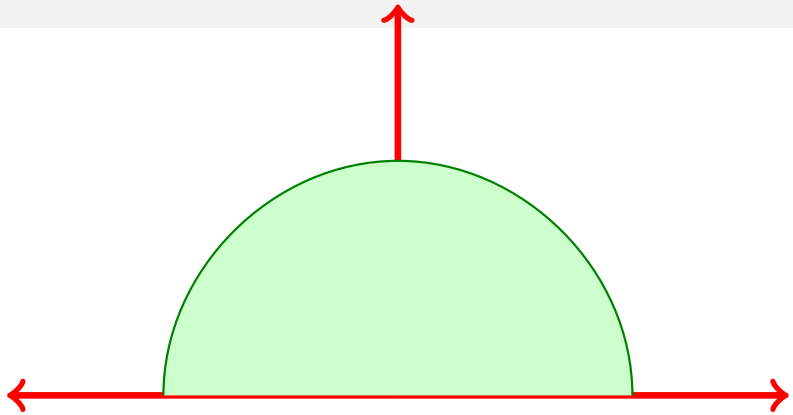

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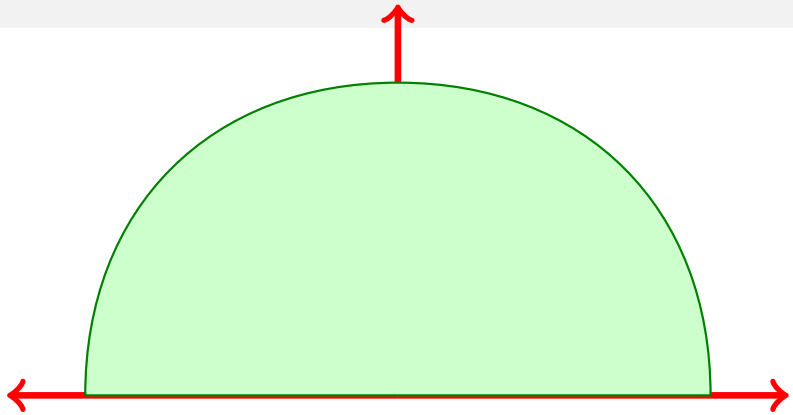

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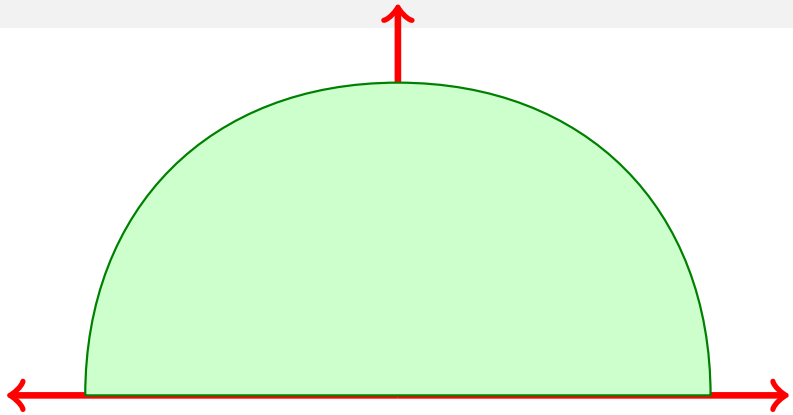

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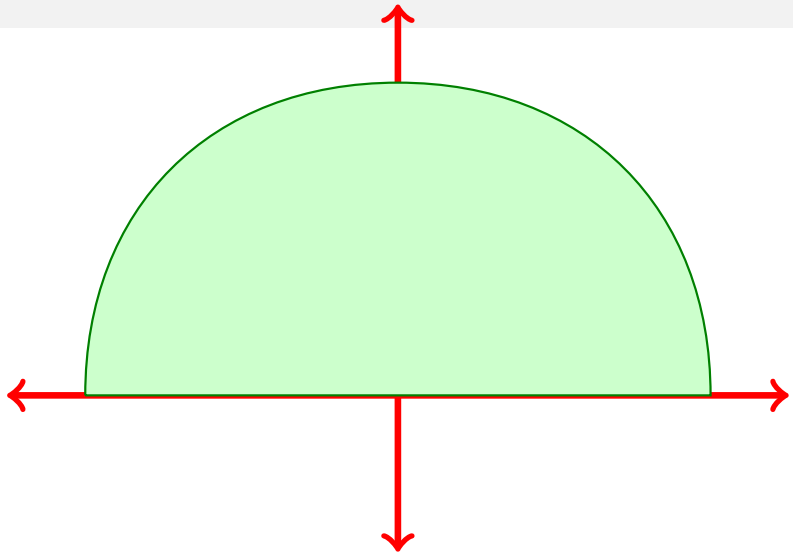

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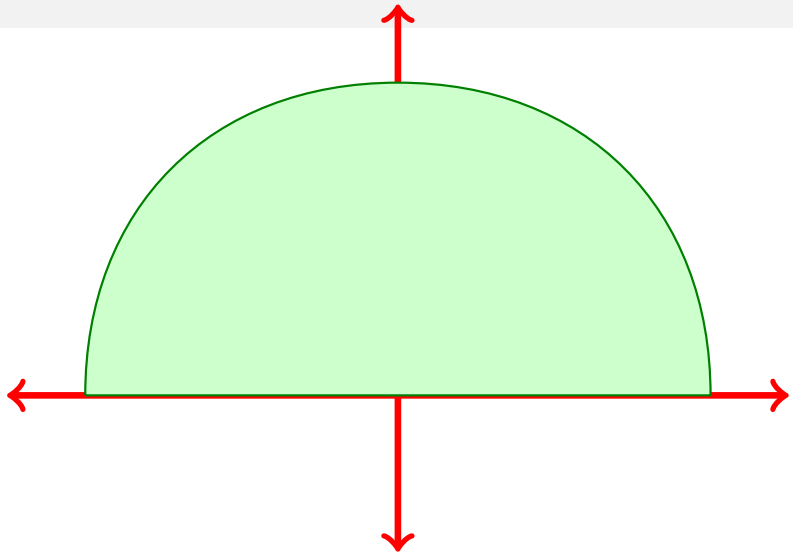

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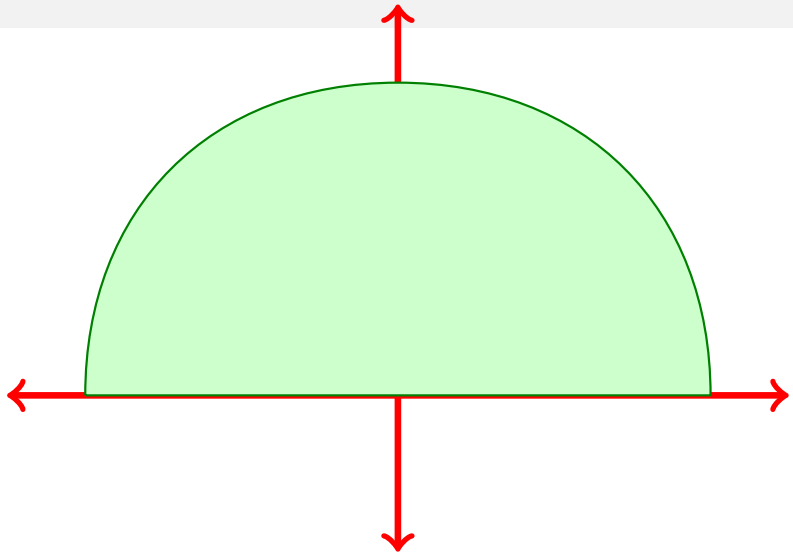

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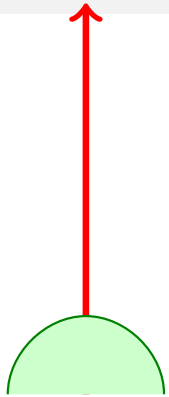

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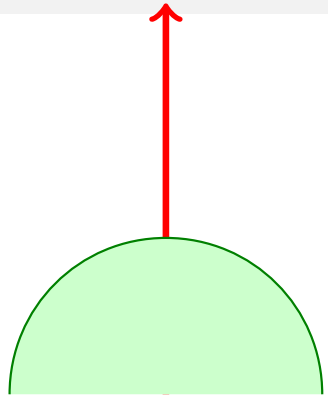

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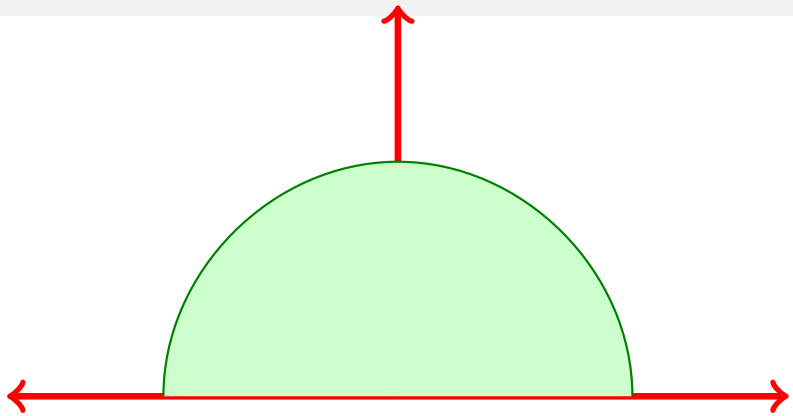

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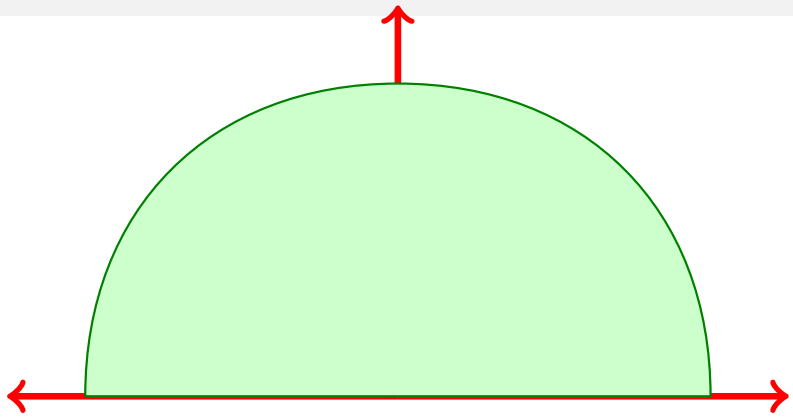

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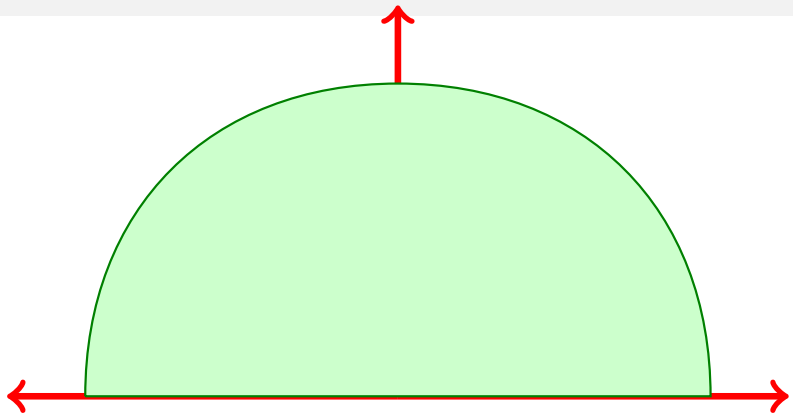

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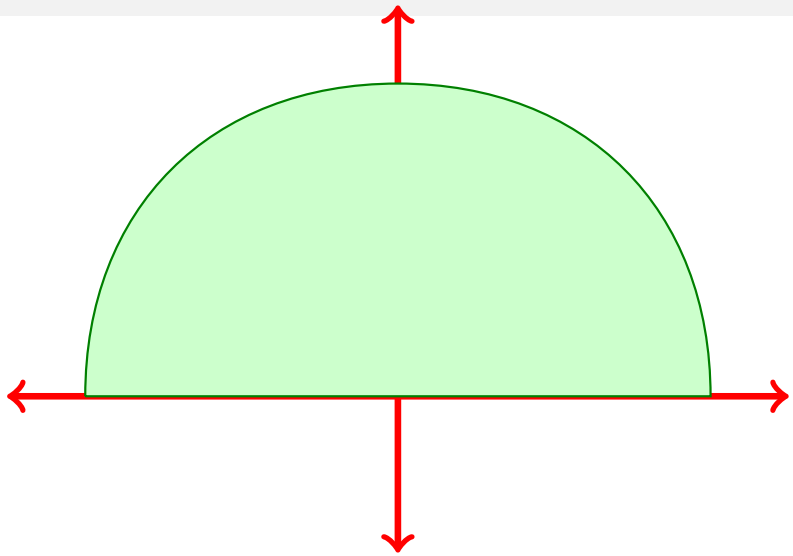

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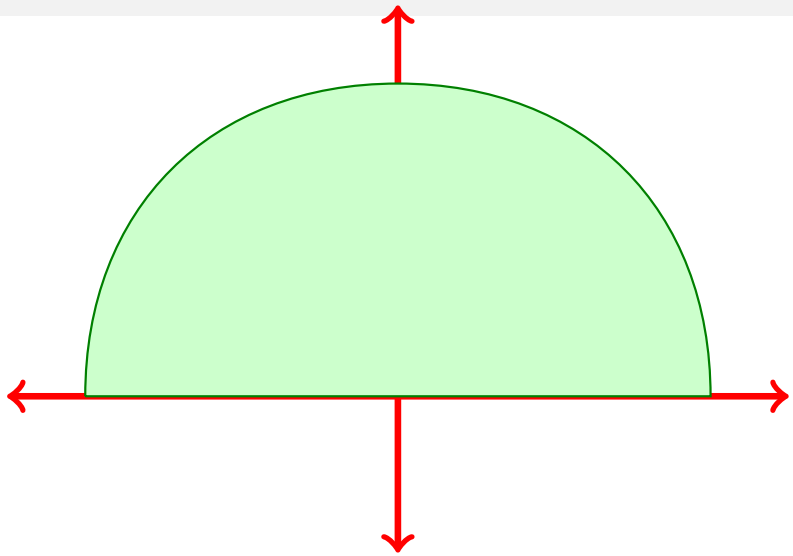

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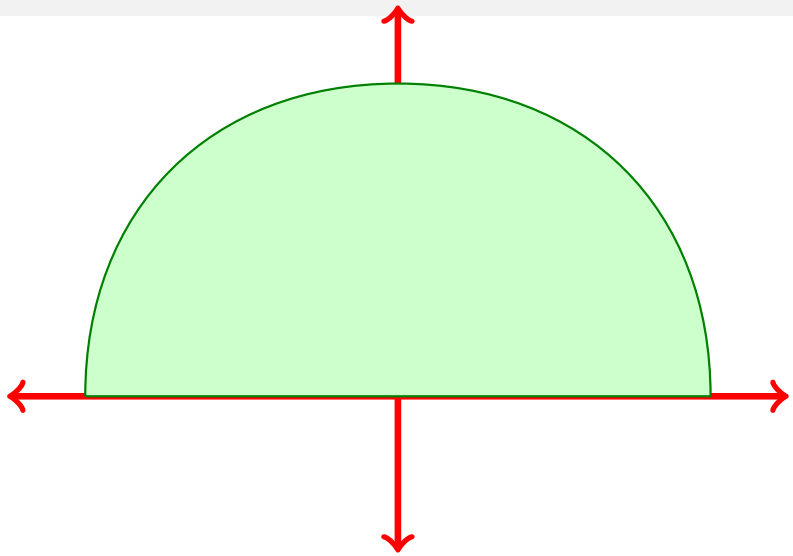

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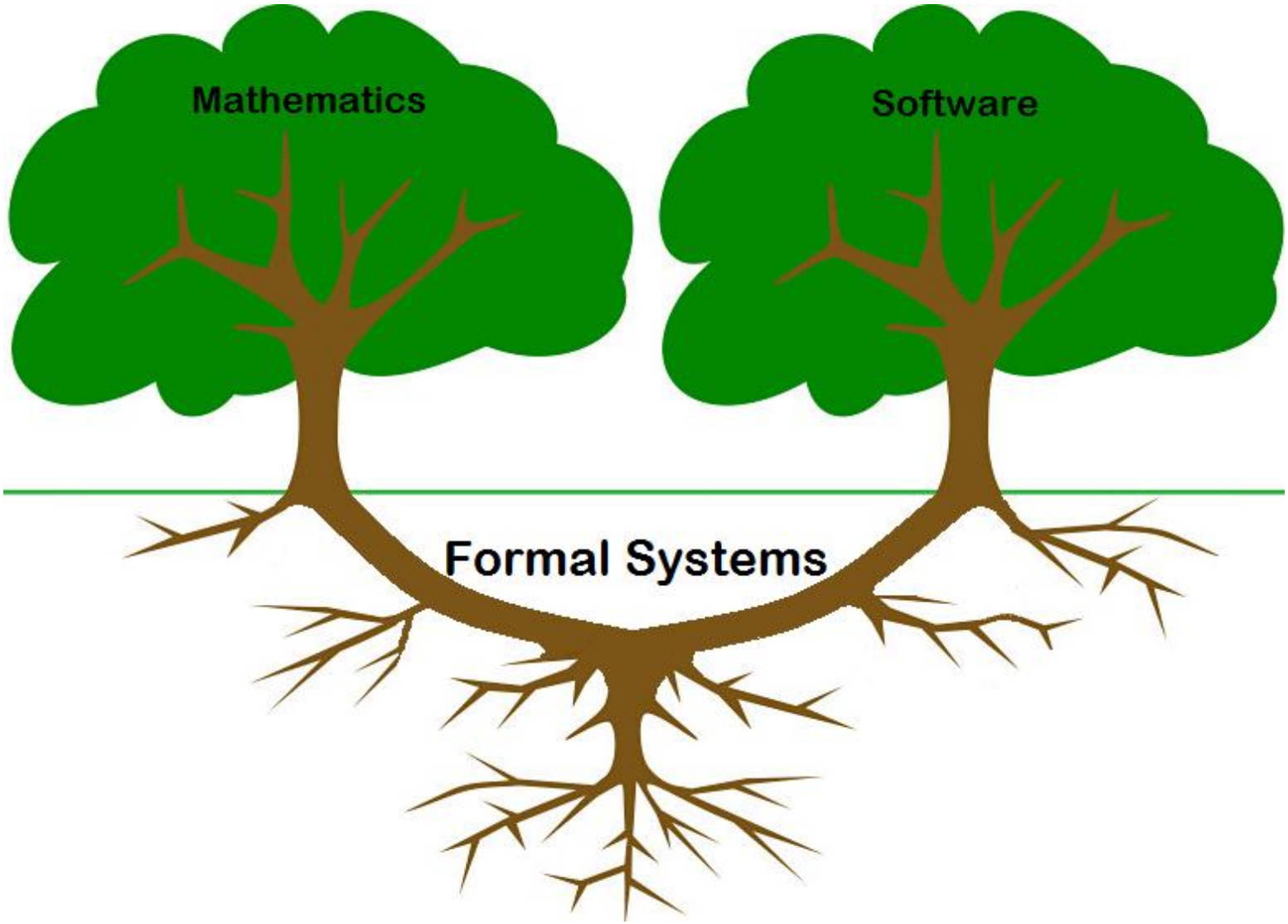
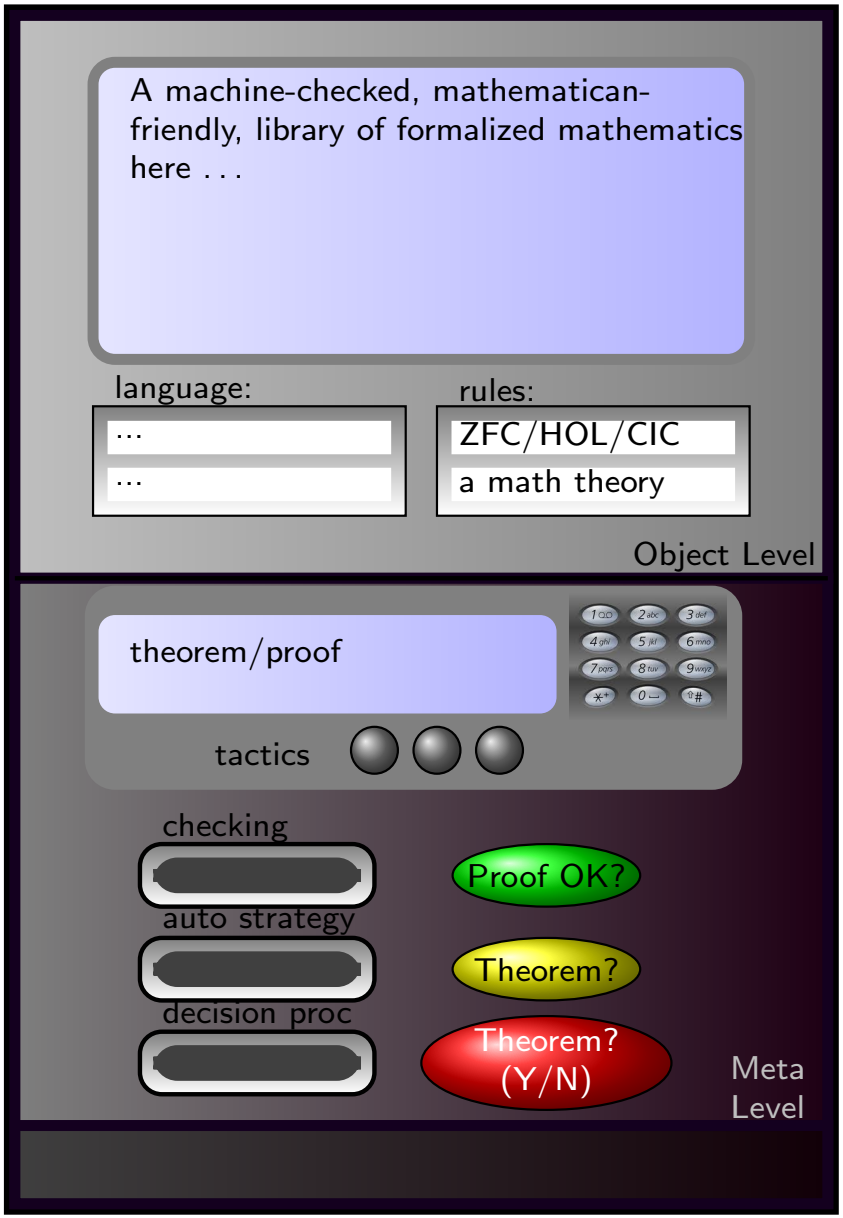
- It continuously grows side-ways, finding more and more fields of application
- Around 1960: Oops, there are difficulties! We now have to go deeper!
- . . . and what do we find?

Common roots



Common future



A quote of a quote

‘ But this self-examination (my note: of mathematics by mathematical means) did have wonderful and totally unexpected consequences in an area far removed from its original goals. It played a big role in the development of the most successful technology of our age, the computer, which after all is just a mathematical machine, a machine for doing mathematics. As E.T. Bell put it, the attempt to soar above mathematics ended in the bowels of a computer! ’

– Gregory Chaitin, The Unknowable, 1999.

Mathematicians and software engineers

Since we refer to these professions often, and ignore all others, we need some clarification. It's easier to define what a mathematician does: mathematics. With software the situation is more complicated, as there are: software engineers, computer systems analysts, developers, programmers, QA engineers, software managers, database designers, etc . . . All of them do software, and with formal methods in mind, these distinctions get in the way.

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So when we refer to 'software engineers' we include all these software professionals. On the other hand, computer scientists and actuaries are included in the mathematicians group. There are many common characteristics between the two professions, some of which are the reasons why they have been ranked in the top 10 best jobs in the past few years.

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But the most interesting common factor is their foundation.

Common roots, but different perspectives

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- There just isn't enough population on the mathematical side

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- Servers or entire networks will go down under targeted attacks

Four views on the gap situation

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My personal experience agrees with item 4. But regardless of my views, the point is that we cannot conceivably accept either 1 or 2; even if they were true, we have to turn things around and move 1 and 2 towards 3 and then on towards 4. The stakes are too high. This program is a small step.

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- Big problem: many Computer Science/Computer Engineeering degrees do not include math requirements for admission and many CS/CE depts do not make up for this during undergraduate studies. The emphasis has been traditionally on the continuous and not on the discrete (because the objective was physics).

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- Following their lead, the program includes a course in 'Discrete Mathematics via Equational Logic'

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- With theorem proving and computer algebra combined, you can see a dazzling display of cooperation between math and software
- We will do this in the 'Computer Algebra Systems' course

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- If the gap is closed, mathematics and software engineering can together lead to better and more powerful tools, which in turn can support more powerful theories, either in mathematics or in software development

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- The first practical language that introduced them was Haskell. Many tutorials explain by different means what a monad is (a container, a computation, etc...) because the thinking is that monads are too mathematical and students won't get them.
- This is not something to be quickly dismissed. *Twisting an abstract mathematical object into something easier to use in engineering is a good intention.*

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We not only cannot avoid categories, *we have to embrace them*. We do that in the advanced sequence of the program, but the basic definitions and examples will be introduced very early. Following is a set of slides that explain this need to become comfortable with categories. They are intended for people who have some familiarity with abstract algebra. Others can safely skip these slides, nothing else in the overview depends on them.

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Removing the operational aspects of a formal system leaves us with an algebraic content that can be studied independently and leads to useful classifications of such systems. Even though these classifications may not give us deep insights into a particular formal system, they give us a very effective way to organize our knowledge so we don't have to keep repeating the same constructions over and over. You may think of categories as the *ultimate pattern recognition tool*.

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- So category theory should feel *natural* to software engineers ...
- ... *because of its high power of abstraction*, not despite of it
- Half-jokingly, the following is referred to as the 'fundamental theorem of software engineering': when things get difficult, raise the abstraction level (term attributed to Koenig)

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In this overview we can only pick one of these items and give some motivation. We pick 'initiality, induction and recursion' because we bump into this in one of the early core courses: 'Formal Semantics of Programming Languages'.

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A directed graph is a set of vertices $A, B, C, \ldots$ *and a set of edges* $f, g, h, \ldots$, *with two functions from edges to vertices, called source and target. If* A *is* $source(f)$ *and* B *is* $target(f)$, *we denote the edge as*

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Source is also called domain, and target codomain. An example of a directed graph:

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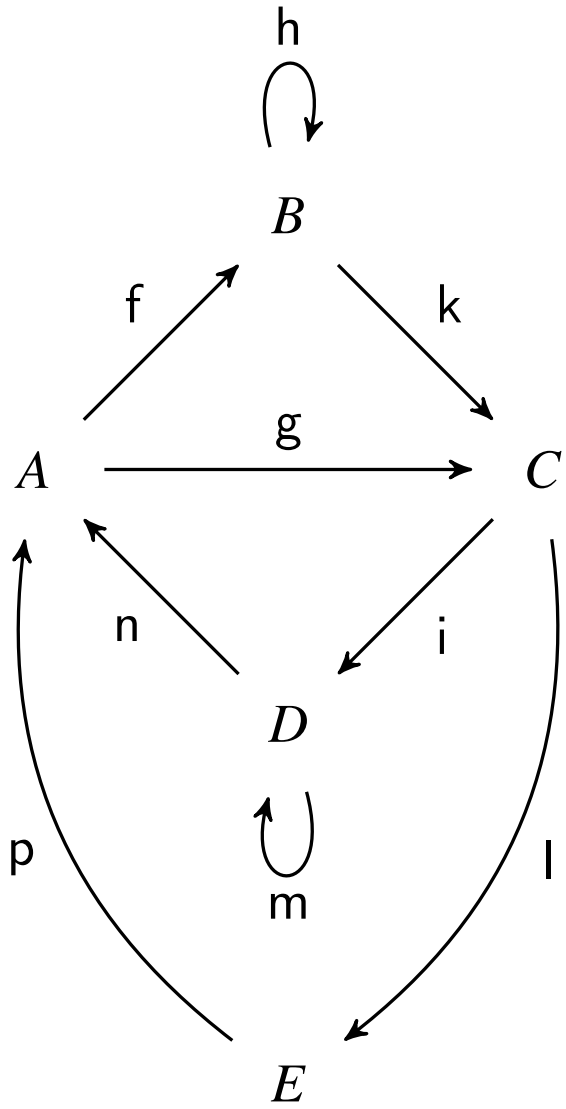

Proof calculi are special graphs

may be skipped

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With proof calculi, the terminology changes, vertices are formulas and edges are proofs.

proof calculus

A proof calculus is a graph satisfying two conditions

- *for each formula* A *there is a special proof* $1_A : A \to A$
- *any two proofs* $f : A \to B$ *and* $g : B \to C$ *can be combined into a proof* $g \circ f : A \to C$

Rules of inference

may be skipped

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These conditions can be written as rules of inference (this explains why they are called 'proof calculi'):

- $\dfrac{}{1_A : A \to A}$ *rlIdentity*
- $\dfrac{f : A \to B \qquad g : B \to C}{g \circ f : A \to C}$ *rlModusPonens*

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A category is a proof calculus where composition of morphisms satisfies two laws

- *For all* $f : A \to B, g : B \to C, h : C \to D$

$$h \circ (g \circ f) = (h \circ g) \circ f \qquad \textit{rlAssociativity}$$

- *For all* $f : A \to B$

$$f \circ 1_A = 1_B \circ f = f \qquad \textit{rlUnit}$$

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With categories the perspective also changes, and the emphasis is now on the equations introduced by the associativity and the unit rules. The only example of a category that we need here: **Sets**, with sets as objects and functions as morphisms (verify that all the rules apply).

Commutative diagrams

may be skipped

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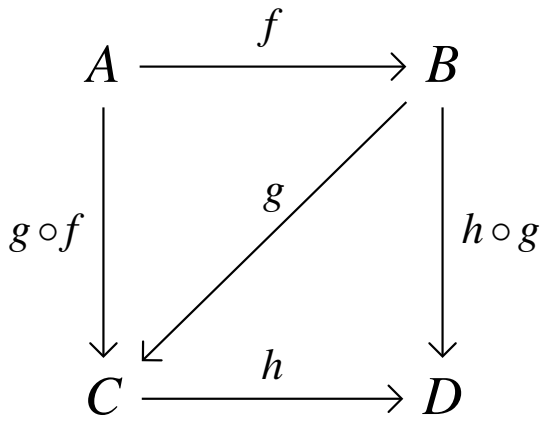
In categories, equations are represented pictorially as commutative diagrams. A diagram is *commutative* if for any two objects in the diagram, all paths leading from one object to the other are equal. For example, the associativity and unit equations are represented by the commutativity of the following two diagrams:

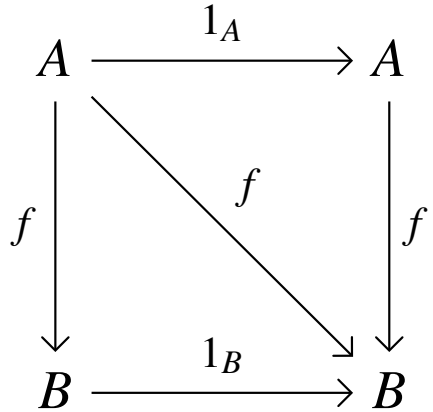
Commutative diagrams

may be skipped

In categories, equations are represented pictorially as commutative diagrams. A diagram is *commutative* if for any two objects in the diagram, all paths leading from one object to the other are equal. For example, the associativity and unit equations are represented by the commutativity of the following two diagrams:



rlAssociativity



rlUnit

Functors

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functor

A functor F between categories **C** *and* **D** *is a map $F : \mathbf{C} \to \mathbf{C}$ taking objects of* **C** *to objects of* **D** *and morphisms $f : A \to B$ of* **C** *to morphisms $Ff : FA \to FB$ of* **D**, *such that*

$$F(g \circ f) = Fg \circ Ff$$

$$F(1_A) = 1_{FA}$$

A functor from a category to itself is called an *endofunctor*.

Polynomial endofunctors

may be skipped

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For two sets A and B, their product $A \times B$ is defined as the set of pairs (x, y) with $x \in X$ and $y \in Y$. Their sum $A + B$ is defined as the disjoint union $\{(x, 1) | x \in A\} \cup \{(y, 2) | y \in B\}$.

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The $\times$ and $+$ operations can be defined more generally for objects A and B of any category through certain 'universal properties' which do not concern us here, except that these universal properties give us two functors: $X \to A \times X$ and $X \to A + X$ for some fixed A; the values on morphisms is given correspondingly. This allows us to build *polynomial endofunctors* on the category **Sets**:

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$$F(X) = C_0 + C_1 \times X + C_2 \times X^2 + \ldots, \text{where } C_n \text{ are fixed sets}$$

Algebraic structures as polynomial endofunctors

may be skipped

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Let's look at monoids, sets with a binary associative multiplication and an identity element. We focus on the signature of the monoid structure, ignoring the monoid axioms. The signature consists of *one constant*, the unit, and *one binary function*, the multiplication, which, if X is a set with a monoid structure, can be written as

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In the category **Sets**, the object 1 stands for a one-element set, $\{*\}$. (By the way, monoids are themselves categories, and the collection of all monoids is also a category, a sign of how broad the concept of category is)

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Instead of looking at these functions individually, we collect them all together into one morphism $1 + X^2 \to X$ in the category **Sets** and therefore obtain that the monoid structure is captured by the polynomial functor:

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algebraic structure	polynomial endofunctor
semigroup	$F_{semigroup}(X) = X^2$
monoid	$F_{monoid}(X) = 1 + X^2$
group	$F_{group}(X) = 1 + X + X^2$
ring	$F_{ring}(X) = 2 + X + 2 \times X^2$

F-algebras

may be skipped

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If F is an endofunctor on a category $\mathbf{C}$*, an F-algebra is a pair* (A, α) *where* A *is an object of* $\mathbf{C}$ *and* α *is a morphism*

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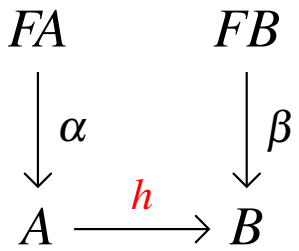

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An object I of a category **C** *is called initial if for any object A of* **C** *there exists a unique morphism $I \to A$. An object T is final if for any object A of* **C** *there exists a unique morphism $A \to T$.*

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- Any two initial objects, when they exist, are isomorphic and any two final objects are isomorphic (easy to check)
- Examples: In **Sets**, the empty set is an initial object and any set consisting of just one element is a final object
- **Most important data types have semantics as initial or final objects in the appropriate category**

Why are initial algebras important?

may be skipped

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may be skipped

Recursive definitions of functions and inductive proofs of their properties are two of the most important concepts in the formalization of software.

What is the relationship between the two? What do they really mean? *Initiality, a categorial concept, gives us the answer*. This is a typical example of why we lean on categories: our program is large and we would not have the time to motivate recursion and induction many times over. We do it once, at the most abstract, conceptual level.

Specifically, the fact that a data type admits an initial algebra semantics means exactly that we can define functions on it *recursively* and we can prove things about these functions *inductively*. Let's look at the simplest case, the data type of natural numbers.

Example: natural numbers as a data type

may be skipped

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Say $f : X \to X$ is a function on an arbitrary set X. Fix an $a \in X$ which is the same thing as fixing a unary function $u : 1 \to X$ where $u(*) = a$. In other words, we have a pair $(X, (u, f) : 1 + X \to X)$, i.e. another F_{nat}-algebra.

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Example: the initial algebra for natural numbers

may be skipped

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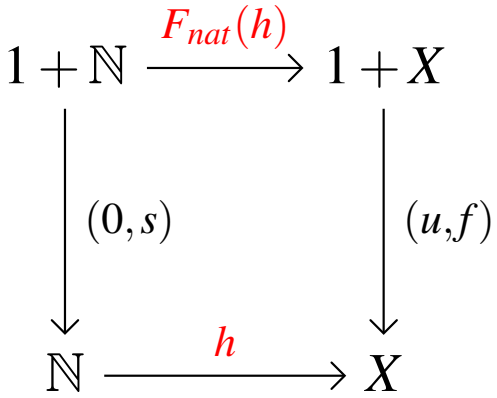
may be skipped

To say that $(\mathbb{N}, (0, s))$ is the initial object in the category of F_{nat}-algebras means, according to the definitions of F-algebras and initiality, that there exists a unique $h : \mathbb{N} \to X$ that makes the following diagram commute:



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Having said that, note that the terms induction and recursion are often interchanged: data can be called inductive or recursive, functions can be called inductive or recursive, etc...

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may be skipped

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So data together with function definitions also form a theory, expressed in the language of Maude. The Maude interpreter calculates (reduces) a program (term) that is expressed in such a theory (which is itself an extension of Maude's equational logic). This view of data and functions as a theory is very useful, not just in Maude but in general.

Example: initial algebra semantics in Maude

may be skipped

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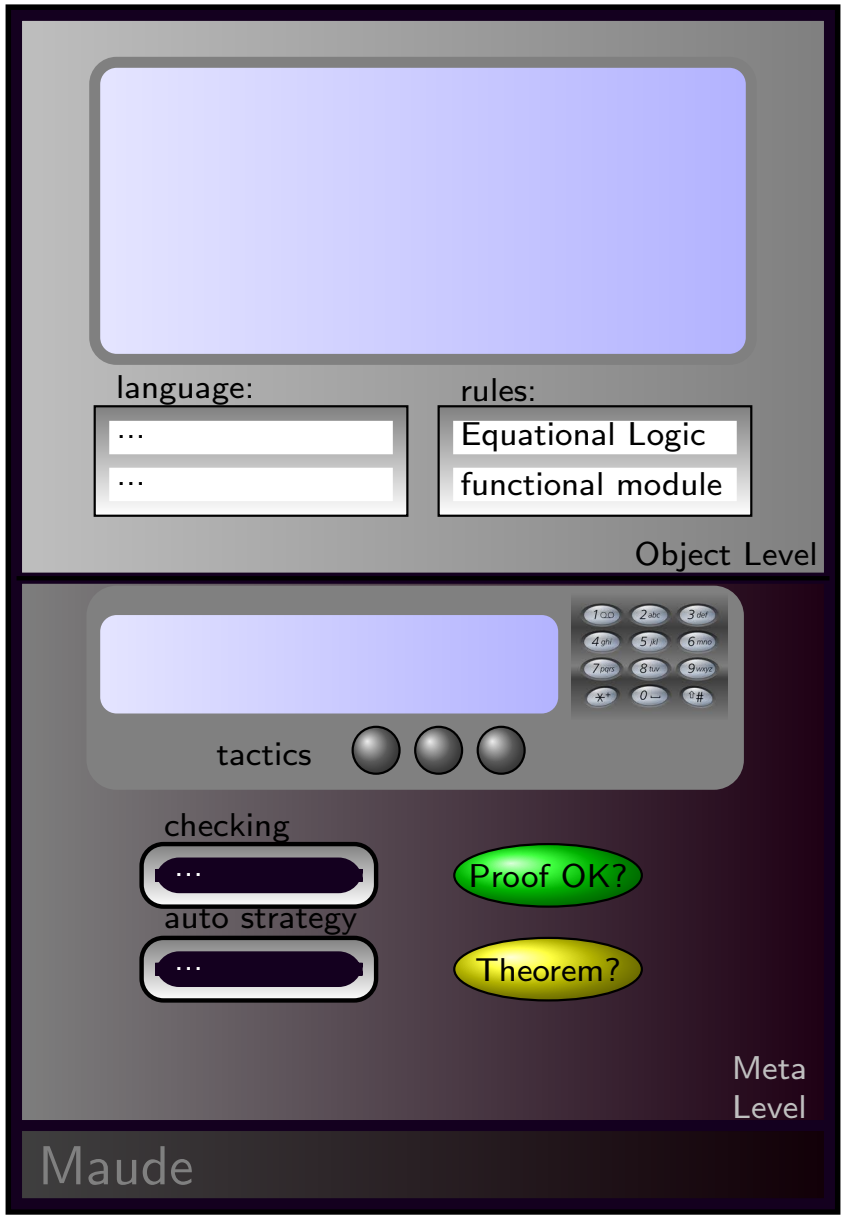
- A Maude functional module defines a theory T in equational logic



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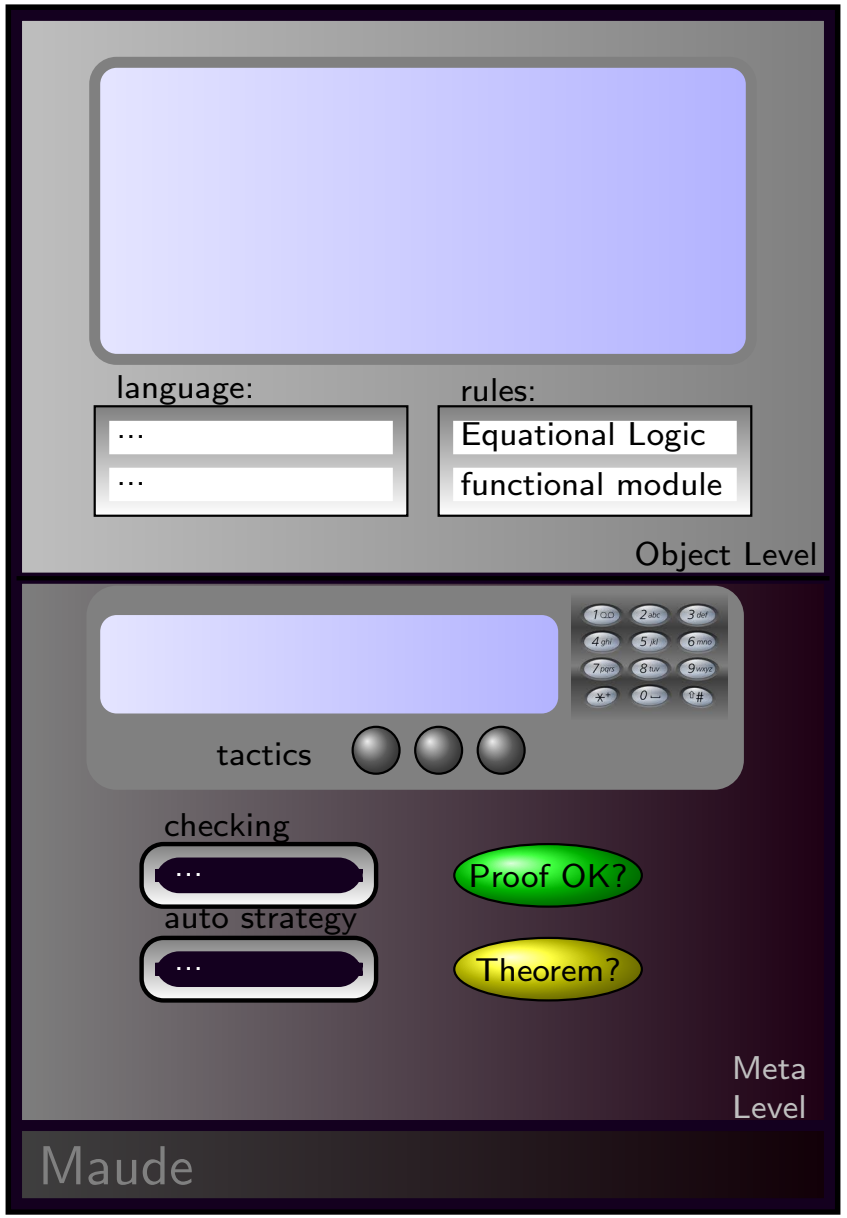

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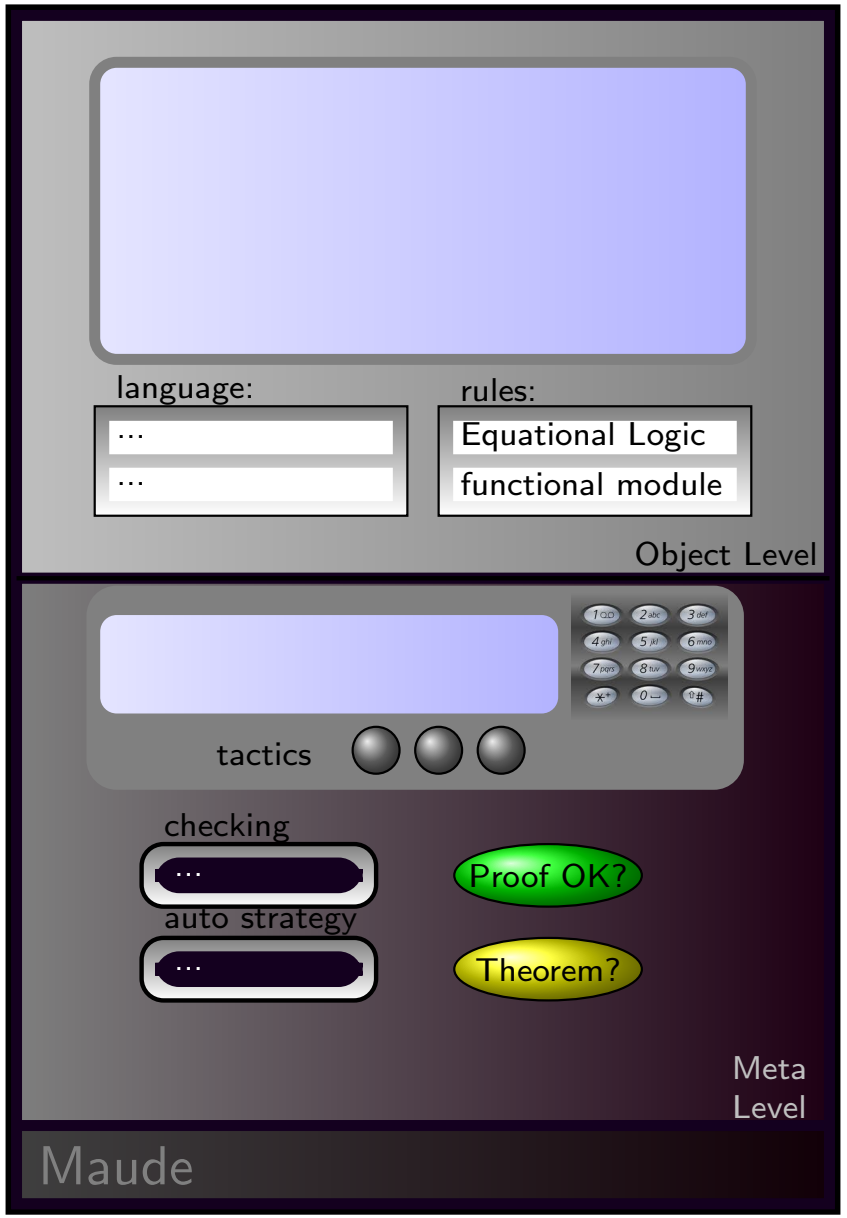

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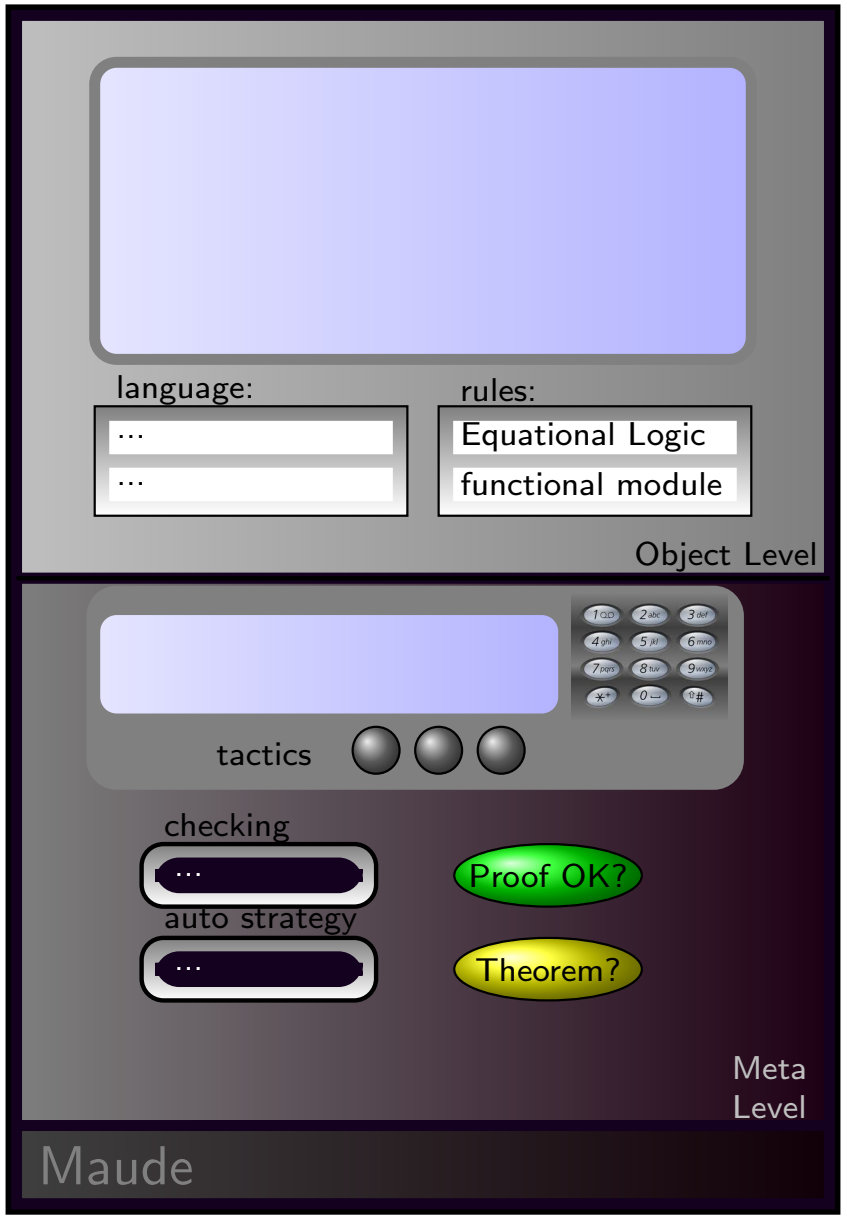
- This construction of the initial algebra of a theory is encountered many times in our program
- (The initial algebra model of propositional calculus is the two-valued Boolean algebra)



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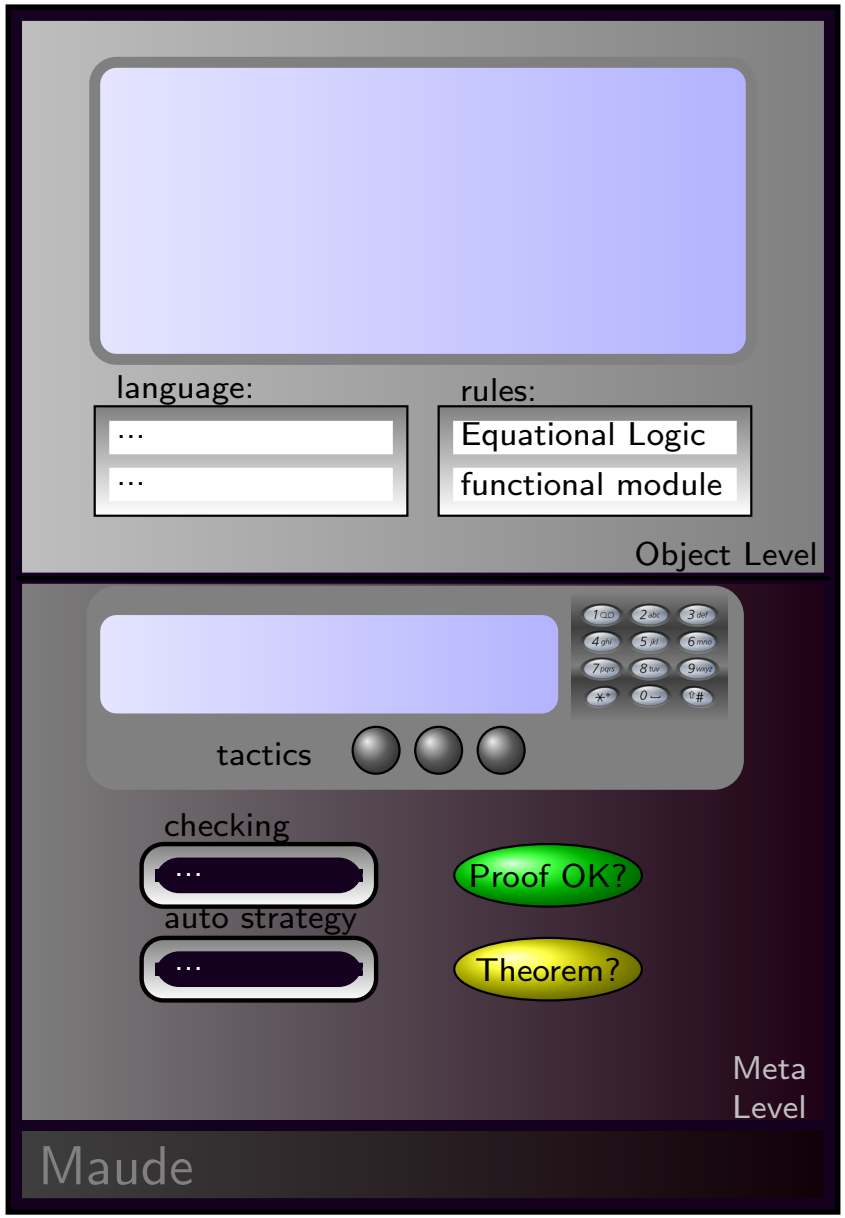

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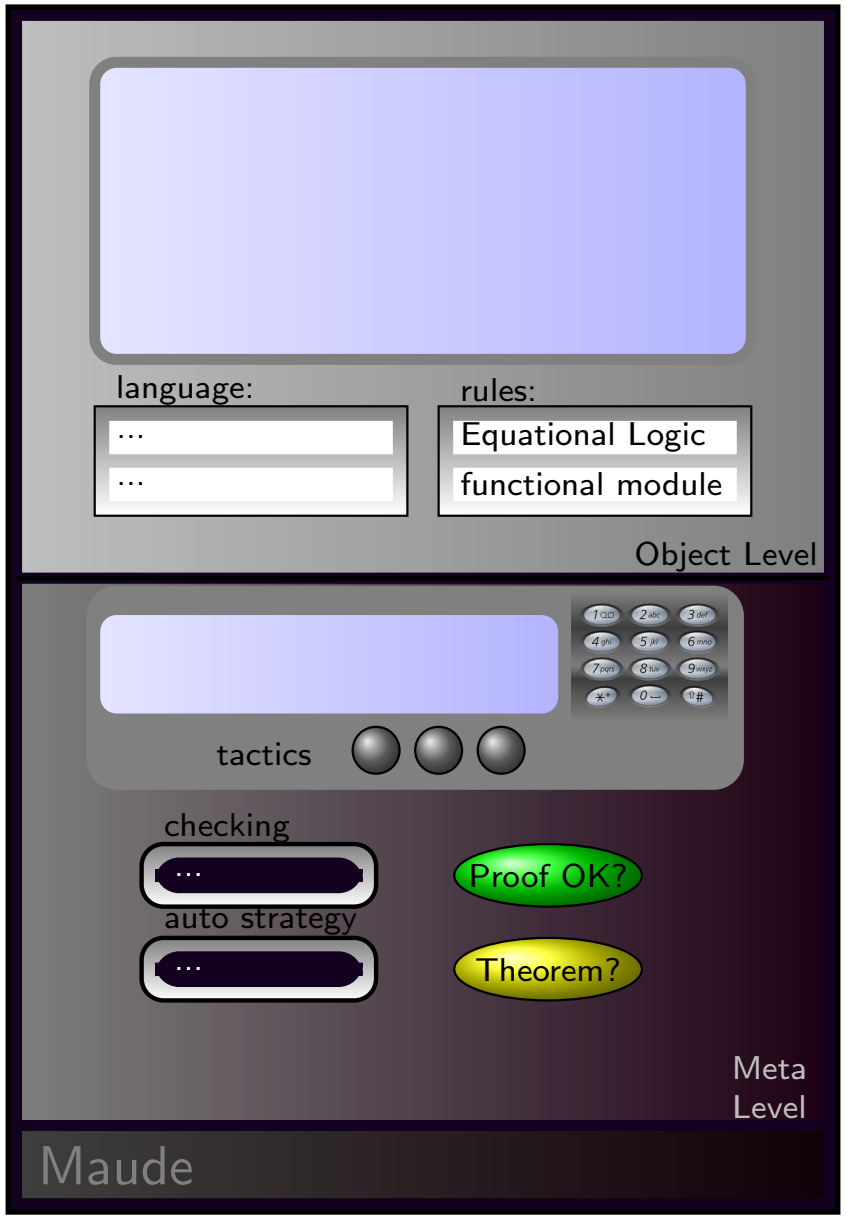

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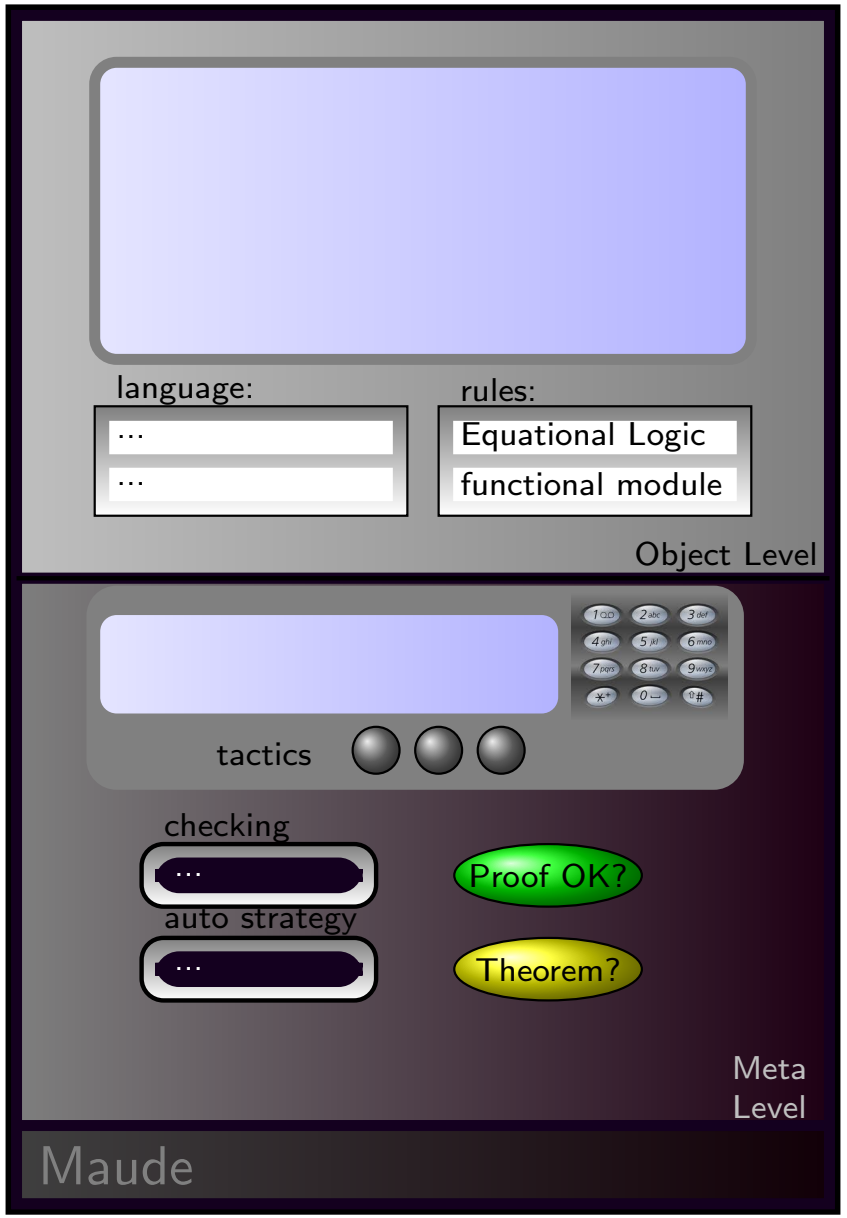

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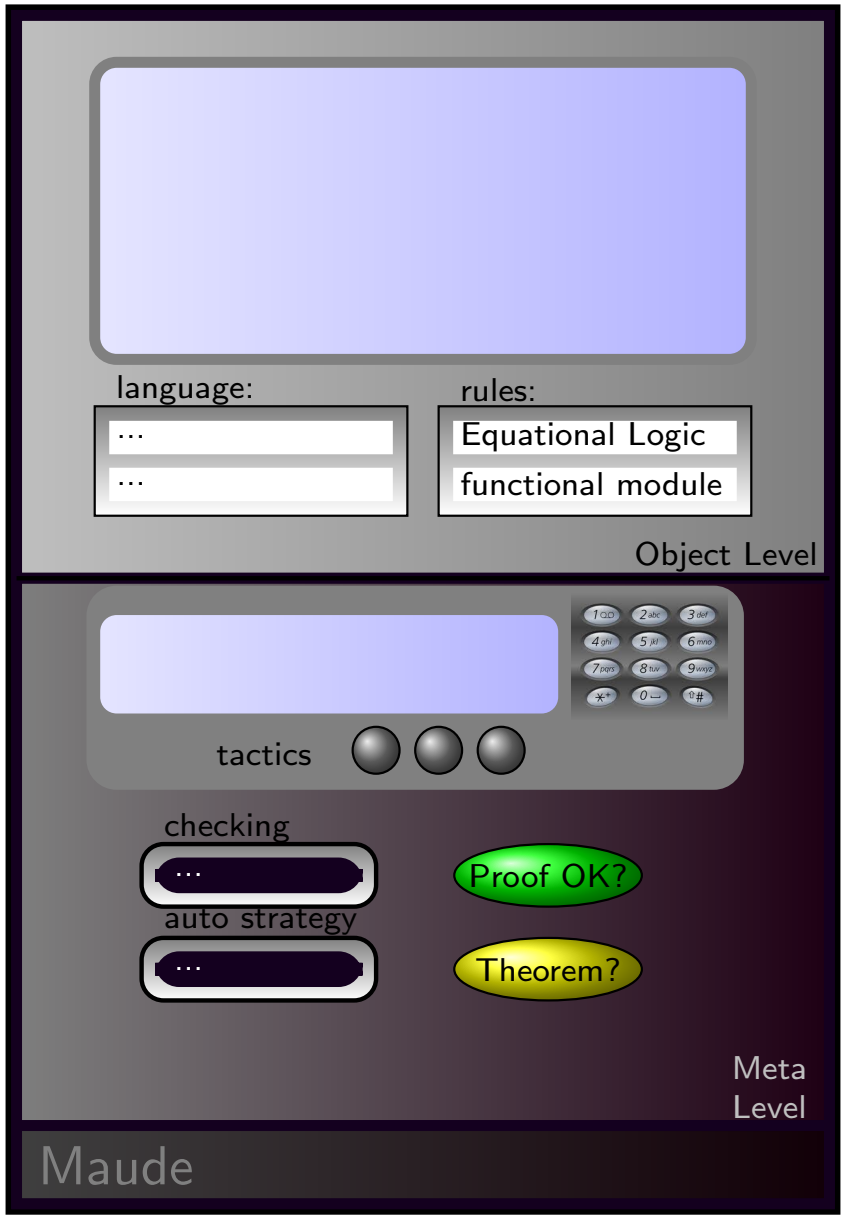

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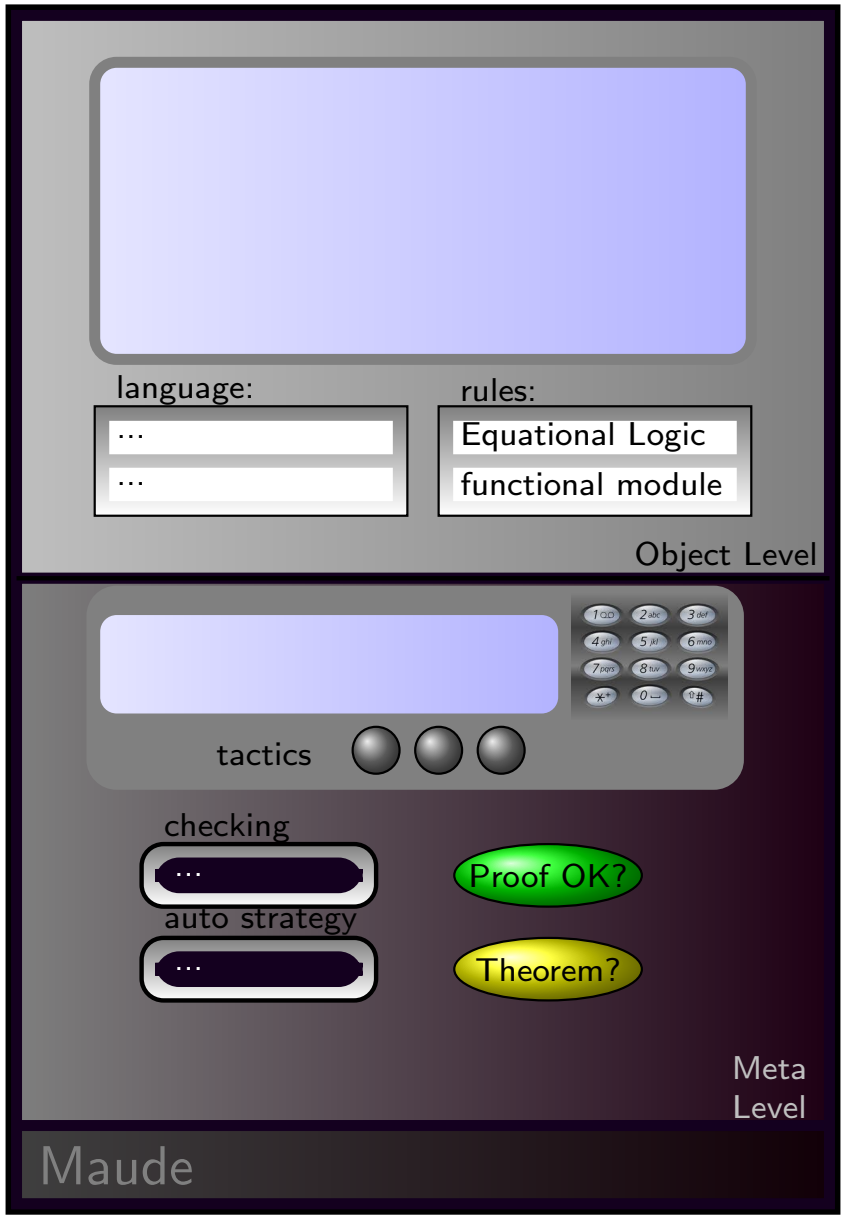
- These two kinds of semantics are basic to everything we do in the program



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The algebraic study of software will only increase. One major piece that is still missing is a convincing algebraic description of the notion of *concurrency*.

Construction versus observation

may be skipped

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may be skipped

It is easier if you look at algebraic types as data, but look at coalgebraic types as systems. So a stream would be a system. We can construct data but we cannot construct systems, we just *observe* systems. We cannot construct a stream, we can just observe it (by observing the head of the stream). We can reason inductively about data but have to reason coinductively about systems. The table below shows this duality.

data and functions	systems and transitions
algebraic type	coalgebraic type
(=inductive type)	(=coinductive type)
(=recursive type)	(=corecursive type)
definition by recursion	definition by corecursion
proof by induction	*proof by coinduction*
type is constructed	type is observed

Why all this algebra language?

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This mention of algebra, coalgebra, categories, etc . . . should not give you the impression that, in the advanced sequence of the program, we'll chase unnecessarily abstract mathematics at the expense of practical applications. Quite the contrary, we need to find a way to speed up towards engineering goals. The truth is that while these applications of category theory are not particularly deep, they do establish a very effective vocabulary for us to use.

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But there is a stronger reason to use categories. Objects of categories can be thought of as 'variable sets'. The mathematics you learned in high-school used set theory and because of this familiarity, it would be nice if we could base everything on ordinary (=constant) sets. It turns out that *many formal systems that are needed in software cannot be modeled with ordinary sets*: e.g. lambda calculus, polymorphism, higher order logic, . . . As it was mentioned earlier, our working (but untested) assumption is that using a higher level of abstraction would eventually feel natural to software engineers.

My setup for lectures

Guest machine: Ubuntu 10.4 64-bit

- Proof General (needs GNU Emacs)
- Maude (logical platform)
- Coq (logical platform)
- Isabelle (logical platform)
- SPIN (model checker)
- LAMP (Apache, MySql and PHP)

Host machine: Windows 7 64-bit

- SWI-Prolog, GHC (Glasgow Haskell Compiler)
- Rodin (platform for Event-B modeling)
- Visual Studio 10

About labs

During the lectures, when you see the following picture . . .

. . . it means that we are branching out of the slide presentation and into a computer session on either Linux or Windows. We call these branch-outs 'labs'.

What follows here is a set of very quick labs (less than 5 minutes each) to give you an idea of the sort of tools that we use in this program and why we use them.

Labs: Examples of formal software development

- *Refining requirements with Event-B*

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- *Building a model of a concurrent system with SPIN*

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- *Solve logical problems with Prolog*

Labs: Examples of formal software development

- *Verify C programs with Why/Frama-C*

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- Introduce formal thinking (and pilot projects) in your organization

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Many of the tools we study are complex systems that have at least a syntactical front-end and a logical back-end implementing a certain logic. You will learn to judge the effectiveness of these tools, their tradeoffs and their suitability for certain practical applications. There are many questions to ask when contemplating formal software development with a specific tool.

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- How complicated are the logical foundations of this proof assistant?

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- Read additional papers before class
- Occasionally make group presentations on selected topics

Learn to estimate the effort to formalize

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Learn to estimate the effort to program in JML or Spec#, rather than Java or C#. Learn to estimate the effort to add verification of safety properties (buffer overflow, arithmetic overflow, null pointer dereference, division by zero) to a C program. Learn to estimate the effort to design a concurrent model of the system that is being built. A example of such estimate would be:

$$N = \text{lines of code}$$
$$\text{theorems} = 2 \times N$$
$$\text{proof lines} = 20 \times \text{theorems}$$

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- Implementation sequence (how the logical tools are built)
- Advanced sequence (a deeper understanding of the theory)

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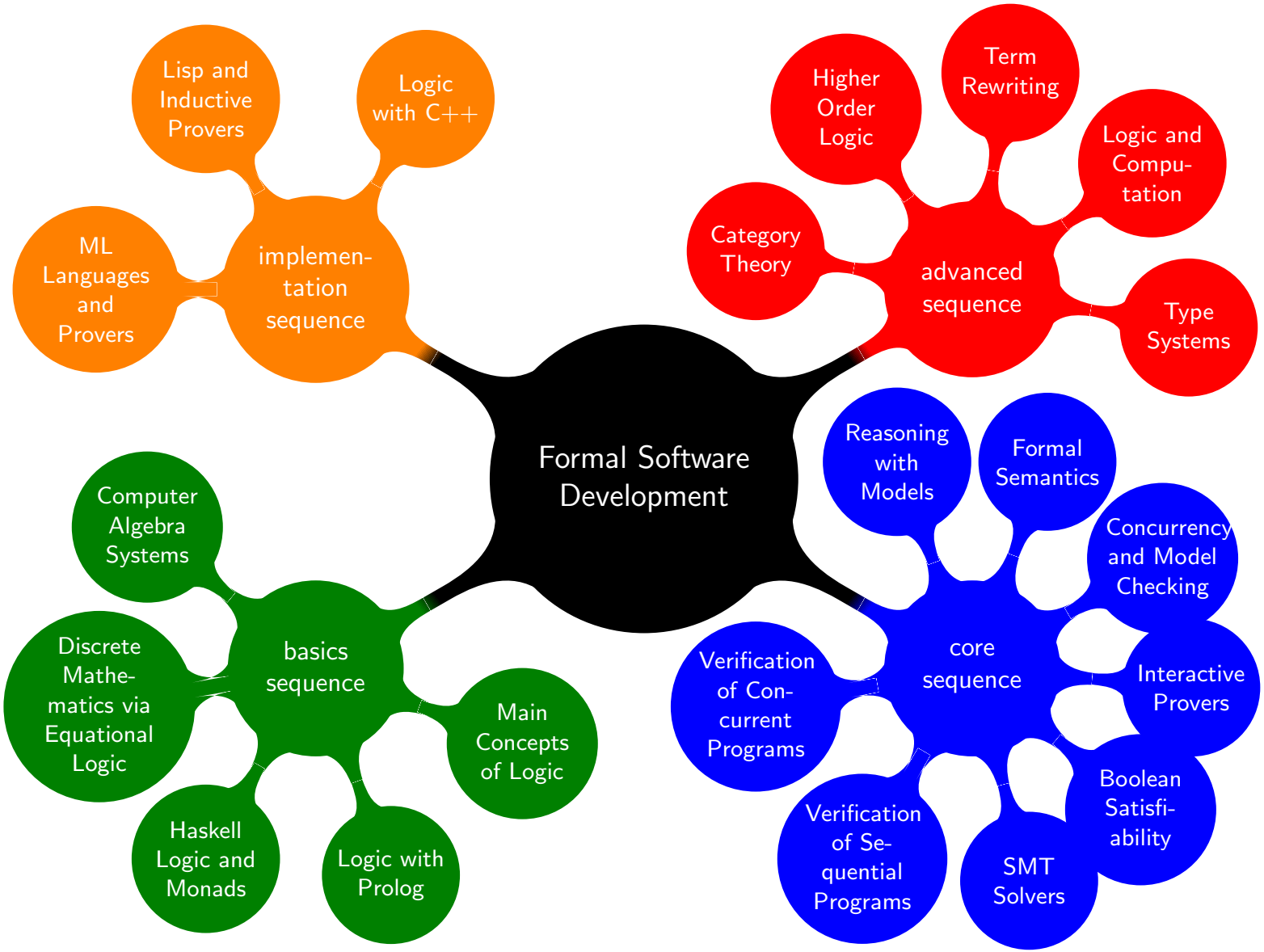

Recommended books I

Jean-Raymond Abrial

Modeling in Event-B System and Software Engineering

Cambridge University Press, 2010.

Manuel Clavel, Francisco Durán, Steven Eker, Patrick Lincoln, Narciso Martí-Oliet, José Meseguer, Carolyn Talcott

All About Maude - A High-Performance Logical Framework

Springer-Verlag, 2007.

Yves Bertot, Pierre Castéran

Interactive Theorem Proving and Program Development: Coq'Art: The Calculus of Inductive Constructions

Springer-Verlag, 2010.

Recommended books II


John Harrison

Handbook Of Practical Logic And Automated Reasoning

Springer-Verlag, 2009.


Gerard J. Holzmann

The SPIN Model Checker: Primer and Reference Manual

Addison-Wesley, 2004.


Tobias Nipkow, Lawrence C. Paulson, and Markus Wenzel

Isabelle/HOL: A Proof Assistant for Higher-Order Logic

Springer-Verlag, 2002.


Krzysztof R. Apt, Frank S. Boer, Ernst-Rüdiger Olderog

Verification of Sequential and Concurrent Programs

Springer-Verlag, 2009.

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- Things are falling into place, as mathematicians and software engineers work from both ends
- *It is up to you how quickly they fall into place!*
- Great time to produce machine-proven software

A big investment on your part

- Formal software development is a huge subject
- It requires a considerable commitment
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  - ...and machine-proven mathematics!

Conclusion

The goals of our program are:

- To begin thinking of software in formal terms
- To deepen our understanding of logic and computation
- To know and use powerful logic tools
- To actively participate in their improvement

Software is . . . stating the right theorems and proving them